Revista Mexicana de Ciencias Agrícolas

DOI: https://doi.org/10.29312/remexca.v16i4.3425

Article

Adjustment to optimal-economic fertilization doses when prices change: economic-mathematical analysis

Adrián González-Estrada[1§]
Antonio Turrent-Fernández[2]

1 Programa Nacional de Economía. Campo Experimental Valle de México-INIFAP. Carretera los Reyes-Texcoco km 13.5, Texcoco, Estado de México, México. CP. 56250

2 Programa Nacional de Maíz. Campo Experimental Valle de México-INIFAP. Carretera los Reyes-Texcoco km 13.5, Texcoco, Estado de México, México. CP. 56250. (turrent.antonio@inifap.gob.mx).

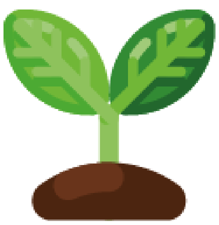
Autor para correspondencia: adrglez@prodigy.net.mx.

Abstract

The current situation shows worrying trends that will hinder both food production and the production and use of fertilizers. By 2050, the world population will be 9 200 million; its demand for food will grow by 54% and for fertilizers by 45% compared to those of 2015. Fertilizer prices have grown considerably faster than agricultural product prices, especially since the first quarter of 2021. Price changes and the need to reduce greenhouse gas emissions modify the optimal-economic doses of fertilization, which is why it is necessary to adjust them. This research aimed to define the economic-mathematical bases for updating the optimal-economic doses of fertilization when the prices of fertilizers and agricultural products change. The method was mathematical-deductive. In addition, the theory of non-linear mathematical programming, the economic theory of profit optimization, the mathematical theorems of the envelope, and the Shephard-McKenzie lemma were applied. The ten propositions obtained constitute the results of this research, and they were obtained through a logical-mathematical process of a deductive nature. It was concluded that the response surfaces should be estimated for each crop and for each of the agricultural regions of Mexico, and it is necessary to create a computerized system that analyzes and updates the optimal-economic doses of fertilization as changes in the prices of outputs and fertilizers are observed.

Palabras clave:

analysis behind the envelope, fertilization dose, input-output price ratio.



Revista Mexicana de Ciencias Agrícolas
REMEXCA
DOI: https://doi.org/10.29312/remexca.v16i4.3425

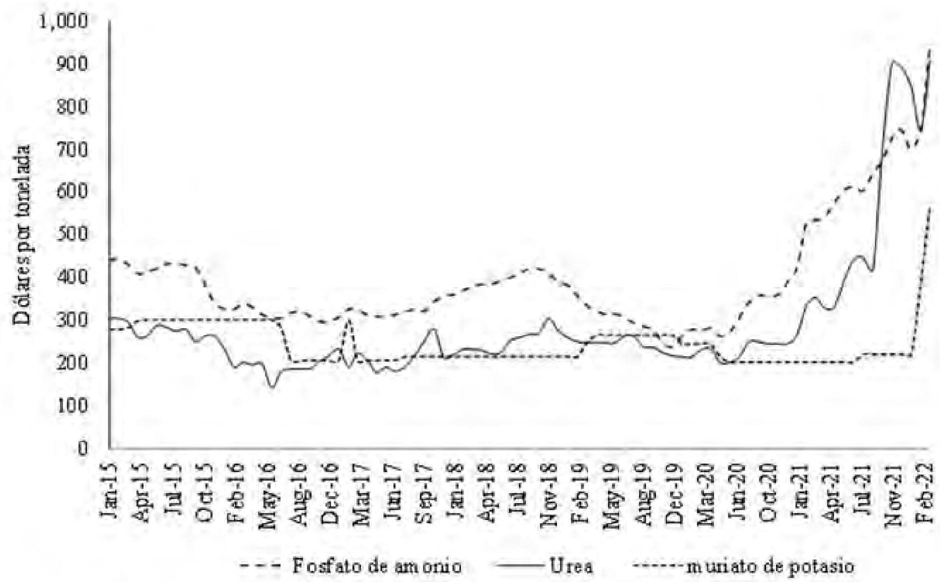
Introduction

Fertilizers are essential agricultural inputs for the sustainable development of agriculture in its intensive phase and for increasing crop yields. Agricultural intensification is one of the dominant trends in agricultural development today and is a necessary, but not sufficient, condition for producing the food required by a growing and developing society. The International Fertilizer Association (IFA, 2018) estimated that without chemical fertilization, global agricultural production would be reduced by 50%.

Borlaug and Doesell (2004) results determined that, by 2050, the world demand for cereals will be 50% higher than in 2000 and that of this increase, 80% will be obtained with the intensification of production processes and 20% with an increase in the cultivated area. Global demand for chemical fertilizers will increase by 45%. On the other hand, González-Estrada (2019) estimated that, by 2050, Mexico will have a population of 162.75 million, its gross domestic product per capita will increase by 72.5%, and the demand for food will grow by 54% compared to that of 2015. This process will require a growing supply of fertilizers.

The study by Heffer and Prud'homme (2008) presented statistical results showing that, at the global level, the input-output ratio between the price of different fertilizers and the price of wheat, rice, and corn increased during the period from 1972 to 2008, which shows that fertilizer prices grew faster than wheat, rice, and corn prices. This secular trend has grown even more in 2021 and 2022 (Banco Mundial, 2022a). As of the first quarter of 2021, fertilizer prices increased exponentially and are expected to increase even more over the next year, as can be seen in Figure 1.

Figure 1. Fertilizer price dynamics, 2015-2022.



According to the Banco Mundial (2022a), in 2021, the average price of fertilizers increased by 90% and so far in 2022, by 30%. On the contrary, in the first quarter of 2022, the price of wheat rose by 30%, corn and soybeans by 20%, and rice by 6%. Fertilizer price growth is expected to continue (World Bank, 2022b). Similar changes have also been observed in Mexico; in 2017, the price of urea was 4.20 \$ kg^{-1}; at the beginning of 2022, urea in Mexico was priced at 28.00 \$ kg^{-1} and in June, at \$18.00 due to subsidies.

DOI: https://doi.org/10.29312/remexca.v16i4.3425

Globally, increases in fertilizer prices have reduced the area planted and yields of major cereals (Siwa and Rosegrant, 2015). In Mexico, not only have the optimal-economic doses of fertilization not been updated, but there is an idea among many farmers and even technicians that they should not change; there has been no official recommendation in this regard.

On the other hand, in most of the research conducted in Mexico, the optimal fertilization doses have been calculated directly from experimental data, without the prior estimation of the response surfaces. Because of this deficiency, updating fertilization rates in the face of such significant changes in fertilizer prices would require each of the researchers who conducted such experiments to calculate the update for each of the more than 100 crops in Mexico and for each of Mexico's 72 agricultural regions (González-Estrada, 1990). A gigantic task that could be shortened if the relevant mathematical and computational algorithms were available.

This research aimed to define the economic-mathematical bases that facilitate the updating of the optimal-economic fertilization doses (OED) when the prices of fertilizers and the crop change. The central hypothesis is that the optimal-economic fertilization doses estimated from experimental data are invalidated after significant changes in input-output ratios occur and therefore should be revised each time these changes occur.

Materials and methods

The method of this research is mathematical-deductive; it is based on the theory of nonlinear mathematical programming, the economic theory of profit optimization, the mathematical theorems of the envelope, and the Shephard-McKenzie lemma (Mas-Colell *et al*., 1995).

It is common practice throughout the world to use quadratic polynomials in research to estimate the response surfaces of crops to different fertilization doses (González-Estrada, 1995). The general classical function that expresses the response of yields (R) to the different applications of nitrogen (N), phosphorus (P) and potassium (K) is:

$R= f(N,P,K) = \beta_0 + \beta_1 N + \beta_2 P+ \beta_3 K+ \beta_4 NP+ \beta_5 NK+ \beta_6 PK- \beta_7 N^2 - \beta_8 P^2 - \beta_9 K^2$

Because in Mexico the response of yields to potassium applications in most crops has been statistically insignificant, the complete polynomial is reduced to:

Where: $\theta= (p_c, p_N, p_P)$ is a vector whose components are parameters that remain fixed in the process of optimizing fertilization doses.

$R= f (N,P,\theta) = \beta_0 + \beta_1 N+ \beta_2 P+ \beta_3 NP- \beta_4 N^{2-} \beta^5 P^2$ 1).

A necessary or indispensable property of these functions is that they are twice continuously differentiable, that they exhibit diminishing yields: $\partial^2 R / \partial N^2 < 0$, $\partial^2 R / \partial P^2 < 0$, that they comply with the Young-Haffer theorem (Sydsaeter *et al*., 2008): $\partial^2 R / \partial N \partial P = \partial^2 R / \partial P \partial N$, and that the response functions are concave (Takayama, 1991 and 1996). It should be noted that this paper defines OED in terms of the active ingredients of N and P and not in terms of commercial products.

Thus, the profit per hectare function estimated with experimental information from fertilization trials is:

$\Pi= (I-C) = p_C (\beta_0 + \beta_1 N+ \beta_2 P+ \beta_3 NP- \beta_4 N^{2-} \beta_5 P^2) - p_N N - p_P P$ 2).

The first-order conditions for finding a critical point are: $p_c (\beta_1 + \beta_3 P-2 \beta_4 N) - p_N = 0$. 3). $p_c (\beta_2 + \beta_3 N-2 \beta_5 P) - p_P = 0$. 4). In matrix form:

$$\begin{pmatrix} -2\beta_4 & \beta_3 \\ \beta_3 & -2\beta_5 \end{pmatrix} \begin{pmatrix} N \\ P \end{pmatrix} = \begin{pmatrix} (p_N - p_C\beta_1)/p_C \\ (p_P - p_C\beta_2)/p_C \end{pmatrix}$$

5).

Since the objective function is concave, then the necessary conditions (3)-(4) are also sufficient to determine the optimal solution. The following optimal-economic doses of fertilization are obtained from system (5):

Revista Mexicana de Ciencias Agrícolas
DOI: https://doi.org/10.29312/remexca.v16i4.3425

$$N^*=\frac{-2\beta_5(p_N-\beta_1)-\beta_3(p_P-p_C\beta_2)}{p_C(4\beta_4\beta_5-\beta_3^2)} \text{ and } P^*=\frac{-2\beta_4(p_P-\beta_2)-\beta_3(p_N-\beta_1)}{p_C(4\beta_4\beta_5-\beta_3^2)}$$

8).

It should be noted that the optimal-economic doses of fertilization are not only the function of the least squares estimators but also depend on the prices of inputs and outputs. The Jacobian matrix of the problem of the function π (N,P) is:

$$I=\begin{pmatrix} \frac{\partial^2\pi}{\partial N^2} & \frac{\partial^2\pi}{\partial N\partial P} \\ \frac{\partial^2\pi}{\partial P\partial N} & \frac{\partial^2\pi}{\partial^2 P} \end{pmatrix}=\begin{pmatrix} -p_C 2\beta_4 N & p_C\beta_3 \\ p_C\beta_3 & -p_C 2\beta_5 \end{pmatrix}$$

The sufficient, second-order conditions that guarantee that the critical point found is a global and unique maximum are: $-2\ p_C\ \beta_4\ N< 0 \wedge (4\ \beta_4\ \beta_5\ N-\ \beta_3\ ^2) > 0$.

In order analyze the changes in the optimal-economic doses of fertilization when input-output prices change, it is necessary to use relative prices (González-Estrada, 1995), whose numerary will be the price of the output obtained in the given crop; that is: $p_c= 1$ and also: $\lambda= (p_N / p_c)$ and $\varphi= (p_P / p_c)$.

Thus, the profit per hectare function is: $\pi= (I\text{-}C) = \beta_0 + \beta_1 N+ \beta_2 P+ \beta_3 NP- \beta_4 N^2- \beta_5 P^2-\lambda N-\varphi P$ 9).

Where: $\beta_i> 0\ \forall\ i= 1, 2, \ldots ,5 \wedge p_c, p_N, p_P,\lambda,\varphi> 0$.

The first-order conditions for finding a critical point are:

$$\begin{pmatrix} -2\beta_4 & \beta_3 \\ \beta_3 & -2\beta_5 \end{pmatrix}\begin{pmatrix} N \\ P \end{pmatrix}=\begin{pmatrix} \lambda-\beta_1 \\ \varphi-\beta_2 \end{pmatrix}$$

10).

Since the objective function is concave, then the necessary conditions (3)-(4) are also sufficient to determine the optimal solution. The optimal-economic fertilization dose (OED) is:

$$N^*=\frac{-2\beta_5(\lambda-\beta_1)-\beta_3(\varphi-\beta_2)}{4\beta_4\beta_5-\beta_3^2}$$

11).

$$\wedge\ P^*=\frac{-2\beta_4(\varphi-\beta_2)-\beta_3(\lambda-\beta_1)}{4\beta_4\beta_5-\beta_3^2}$$

12).

The last stage of the method of this research consists of applying the envelope theorems to expressions (6), (7), (11) and (12). According to González-Estrada (2022), the objective function in unrestricted optimization problems is defined in terms of a set of n independent variables, which, when evaluated at a critical point x*, result in f(x*).

This optimal value of y= f(x) is a function of the k parameters: $\theta_1, \theta_2,\ldots,\theta_k$, of the problem; that is: $y^* = f(x^*,\theta) = f(x_1^*, x_2^*,\theta,x_n^*; \theta_1, \theta_2,\ldots,\theta_k)$, where $x=(x_1, x_2,\ldots,_{xn}) \in X\subset R^n$ y $\theta= (\theta_1, \theta_2,\ldots, \theta_k)\in R^k$.

In addition, the optimal vector, x* (θ), is also a function of the parameter vector, which is why: $f^* (\theta) = f (x^*(\theta); \theta)$. The answer to the question of how $f^*(\theta)$ changes when one of the parameters θ_j changes is found in the envelope theorem.

According to González-Estrada (2022), in optimization problems with equality constraints, the objective function is defined in terms of a set of n independent variables, which, when evaluated at a critical point x*, result in L (x*, λ*) and f (x*, λ*). These optimal values of the Lagrangean of the objective function and of the choice variables are a function of the *k* parameters: $\theta_1, \theta_2,\ldots,\theta_k$, of the problem; that is: $y^* = f (x^*, \lambda^*; \theta= f (x_1^*, x_2^*,\lambda, x_n^*, \lambda_1^*, \lambda_2^*,\ldots, \lambda_m^*; \theta_1, \theta_2,\ldots, \theta_k) = f^*(\theta_1, \theta_2,\ldots,\theta_k)$ and $L^* = L (x^*, \lambda^*; \theta) = L (x_1^*, x_2^*,\ldots, x_n^*, \lambda_1^*, \lambda_2^*,\ldots,\lambda_m^*; \theta_1, \theta_2,\ldots,\theta_k) = L^* (\theta_1, \theta_2,\ldots, \theta_k)$. Where: $x= (x_1, x_2,\ldots, x_n)\in X\subset R^n$, $\lambda= (\lambda_1^*,\ldots, \lambda_m^*) \in R^m$ y $\theta= (\theta_1, \theta_2,\ldots,\theta_k) \in R^k$.

DOI: https://doi.org/10.29312/remexca.v16i4.3425

Note that x*(θ) is also a function of the parameter vector, which is why: f * (θ) = f (x * (θ), λ * (θ); θ). The answer to the question of how f * (θ) and x * (θ) change in the face of a change in one of the parameters θ_j is found in the following theorem (González-Estrada, 2022).

Envelope theorem

If the functions f (x,λ; θ), f (x* (θ), λ* (θ); θ) and x* (θ) are continuously differentiable and if f* (θ) = f (x*, λ*; θ) = Max f (x,θ) for x= ∈X, and x* (θ) is a critical point, then:

$$\frac{\partial f^{*}(\theta)}{\partial \theta_j}=\frac{\partial f\left(x^{*}(\theta),\lambda^{*}(\theta);\theta\right)}{\partial \theta_j}=\left.\frac{\partial f(x,\lambda;\theta)}{\partial \theta_j}\right]_{x=x^{*}(\theta)}$$

On the other hand, the change of the optimal solution, x*(θ), in the event of a change in any of the parameters is simply: $\partial x^{*}(\theta)\, \partial / \theta_j$).

Results and discussion

According to (6)-(7), the optimal-economic fertilization doses are neither invariant nor fixed, but rather changing. It is true that the natural relationships expressed in the experimental data and in least squares estimators of response surfaces can be considered constant in the short and medium term, but not in the long term. However, the optimal-economic doses of fertilization are changing because they depend on the variation of economic conditions, which, in this case, are expressed in the input-output price ratios λ and φ.

Proposition 1

The OED of nitrogen and phosphorus increase as the price of the output obtained in the crop increases; that is:

$$\frac{\partial N^{*}}{\partial p_C}>0;\frac{\partial P^{*}}{\partial p_C}>0$$

13).

Demonstration (by construction). According to the envelope theorem applied to (6) and (7):

$$\frac{\partial N^{*}}{\partial p_C}=\frac{p_C\beta_2\beta_3\left(4\beta_4\beta_5-\beta_3^2\right)-\left(4\beta_4\beta_5-\beta_3^2\right)\left[-2\beta_5\left(p_N-\beta_1\right)-\beta_3\left(p_P-p_C\beta_2\right)\right]}{p_C\left(4\beta_4\beta_5-\beta_3^2\right)^2}$$

On the other hand,

$$\frac{\partial N^{*}}{\partial p_C}=\frac{p_C\beta_2+2\beta_5\left(p_N-\beta_1\right)+\beta_3\left(p_P-p_C\beta_2\right)}{p_C\left(4\beta_4\beta_5-\beta_3^2\right)}>0$$

14).

Since, according to the conditions of sufficiency: pC (4β4 β5 - β32) > 0; also: $p_C\ \beta_2\ \beta_3$ is positive; $2\beta_5\ (p_N - \beta_1) > 0$, $(p_P - p_C\ \beta_2) > 0 \Rightarrow \beta_3\ (p_P - p_C\ \beta_2) > 0$. On the other hand:

$$\frac{\partial P^{*}}{\partial p_C}=\frac{-\left(4\beta_4\beta_5-\beta_3^2\right)\left[-2\beta_4\left(p_P-\beta_2\right)-\beta_3\left(p_N-\beta_1\right)\right]}{p_C\left(4\beta_4\beta_5-\beta_3^2\right)^2}$$

$$\therefore\frac{\partial P^{*}}{\partial p_C}=\frac{2\beta_4\left(p_P-\beta_2\right)+\beta_3\left(p_N-\beta_1\right)}{p_C\left(4\beta_4\beta_5-\beta_3^2\right)}>0$$

15).

Since: $2\beta_4\ (p_P - \beta_2) > 0$ y $\beta_3\ (p_N - \beta_1) > 0$ and $p_C\ (4\beta_4\ \beta_5 - \beta_3{}^2) > 0$.

Revista Mexicana de Ciencias Agrícolas
REMEXCA
DOI: https://doi.org/10.29312/remexca.v16i4.3425

Proposition 2

The optimal-economic dose of nitrogen decreases as the price of nitrogen increases and declines with rises in the price of phosphorus; that is:

$$\frac{\partial N^*}{\partial p_N} < 0 \frac{\partial N^*}{\partial p_P} < 0$$

16).

Demonstration (by construction).

$$\frac{\partial N^*}{\partial p_N} = \frac{-2\beta_5 p_C\left(4\beta_4\beta_5 - \beta_3^2\right)}{\left[p_C\left(4\beta_4\beta_5 - \beta_3^2\right)\right]^2} = -\frac{2\beta_5}{p_C\left(4\beta_4\beta_5 - \beta_3^2\right)} < 0$$

17).

$$\frac{\partial N^*}{\partial p_P} = -\frac{\beta_3 2\beta_5\left(p_N - \beta_1\right)}{\left[p_C\left(4\beta_4\beta_5 - \beta_3^2\right)\right]^2} < 0$$

18).

Proposition 3

The optimal-economic dose of phosphorus decreases as the price of phosphorus increases and declines with rises in the price of nitrogen; that is:

$$\frac{\partial P^*}{\partial p_P} < 0 \frac{\partial P^*}{\partial p_N} < 0.$$

16).

Demonstration (by construction). After applying the envelope theorem to (6) and (7), the following is obtained:

$$\frac{\partial P^*}{\partial p_P} = \frac{-2\beta_4\left[p_C\left(4\beta_4\beta_5 - \beta_3^2\right)\right]}{\left[p_C\left(4\beta_4\beta_5 - \beta_3^2\right)\right]^2} = -\frac{2\beta_4}{p_C\left(4\beta_4\beta_5 - \beta_3^2\right)} < 0$$

20)

$$\frac{\partial P^*}{\partial p_N} = \frac{-\beta_3\left[p_C\left(4\beta_4\beta_5 - \beta_3^2\right)\right]}{\left[p_C\left(4\beta_4\beta_5 - \beta_3^2\right)\right]^2} = -\frac{\beta_3}{p_C\left(4\beta_4\beta_5 - \beta_3^2\right)} < 0$$

21)

So far, the partial effects that changes in individual prices have on optimal-economic fertilization doses have been analyzed. Nonetheless, in reality, there may be simultaneous variations of different magnitudes in the price of the output and in those of fertilizers, which is why the relative prices change and, consequently, the optimal-economic doses of fertilization change.

In addition, in the current situation, increases in fertilizer prices accompanied by increases in prices are observed, which does not necessarily mean that the effects of these changes are counteracted and that they do not modify the optimal-economic fertilization doses. In the analysis that follows, it is shown that absolute changes in output prices and changes in fertilizer prices are not relevant. The correct thing to do is to analyze the changes in the optimal fertilization doses in terms of the relative changes in the input-output price indices: $\lambda = (p_N / p_c)$ y $\varphi = (p_P / p_c)$.

Revista Mexicana de Ciencias Agrícolas

DOI: https://doi.org/10.29312/remexca.v16i4.3425

Proposition 4

The optimal-economic dose of nitrogen decreases as input-output prices increase; that is:

$$\frac{\partial N^*}{\partial \lambda} < 0; \qquad \frac{\partial N^*}{\partial \varphi} < 0$$

26).

Demonstration. From the application of the envelope theorem to (11) and (12), the following is obtained:

$$\frac{\partial N^*}{\partial \lambda} = \frac{-2\beta_5\left(4\beta_4\beta_5 - \beta_3^2\right)}{\left[4\beta_4\beta_5 - \beta_3^2\right]^2} = -\frac{2\beta_5}{4\beta_4\beta_5 - \beta_3^2} < 0$$

27).

$$\frac{\partial N^*}{\partial \varphi} = \frac{-\beta_3\left(4\beta_4\beta_5 - \beta_3^2\right)}{\left[4\beta_4\beta_5 - \beta_3^2\right]^2} = -\frac{\beta_3}{4\beta_4\beta_5 - \beta_3^2} < 0$$

28).

Proposition 5

The optimal-economic dose of phosphorus decreases as input-output prices increase; that is:

$$\frac{\partial \mathrm{P}^*}{\partial \lambda} < 0\frac{\partial \mathrm{P}^*}{\partial \varphi} < 0$$

19).

Demonstration. From (11), (12), and the application of the envelope theorem, the following is obtained:

$$\frac{\partial \mathrm{P}^*}{\partial \lambda} = \frac{-\beta_3\left(4\beta_4\beta_5 - \beta_3^2\right)}{\left[4\beta_4\beta_5 - \beta_3^2\right]^2} = -\frac{\beta_3}{4\beta_4\beta_5 - \beta_3^2} < 0$$

30).

$$\frac{\partial P^*}{\partial \varphi} = \frac{-2\beta_4\left(4\beta_4\beta_5 - \beta_3^2\right)}{\left[4\beta_4\beta_5 - \beta_3^2\right]^2} = -\frac{2\beta_4}{4\beta_4\beta_5 - \beta_3^2} < 0$$

31).

Proposition 6

The effect of a change in the relative price of phosphorus on the optimal dose of nitrogen is equal to the effect of a change in the relative price of nitrogen on the OED of potassium:

$$\frac{\partial \mathrm{N}^*}{\partial \varphi} = \frac{\partial \mathrm{P}^*}{\partial \lambda}$$

32).

Obvious demonstration from (28) and (30). A kind of reciprocity condition (Chiang and Wainwright, 2012) is that obtained from (32).

DOI: https://doi.org/10.29312/remexca.v16i4.3425

Proposition 7 (symmetry of cross-effects)

The cross-effect that changes in the phosphorus/output price ratio have on profits through the optimal nitrogen dose is equal to those that changes in nitrogen/output prices have on profits through the optimal dose of phosphorus:

$$\frac{\partial^2\pi}{\partial N^*\partial\varphi}=\frac{\partial^2\pi}{\partial P^*\partial\lambda}$$

Proposition 8

Increases in fertilizer prices, whether absolute or relative, decrease farmers' profits, whereas increases in the prices of the output obtained increase them:

$$\frac{\partial\pi^*}{\partial p_N}<0\frac{\partial\pi^*}{\partial p_P}<0\frac{\partial\pi^*}{\partial\lambda}<0\frac{\partial\pi^*}{\partial\varphi}<0\frac{\partial\pi^*}{\partial p_C}>0$$

33). (obvious demonstration).

In order to know how much profits change when the prices of the output and fertilizers change, Hotelling's lemma was applied, which is found in Varian (1992); Mas-Colell *et al*. (1995); Jehle and Reny (2011) and the following proposition was obtained.

Proposition 9

In the vicinity of the optimal solution, a unit increase in the price of the output increases the profits by R * pesos, whereas a unit increase in the price of nitrogen or phosphorus decreases the profits N * and P * pesos, respectively:

$$\frac{\partial\pi^*}{\partial p_C}=R^*;\frac{\partial\pi^*}{\partial p_N}=-N^*;\frac{\partial\pi^*}{\partial p_P}=-P^*$$

34).

Demonstration. Since:

$\pi^*(p_C, p_N, p_P) = p_C R^*(p_C, p_N, p_P) - p_N N^*(p_C, p_N, p_P) - p_P P^*(p_C, p_N, p_P)$,

then, according to the envelope theorem (Simon and Blume, 1994):

$$\frac{\partial\pi^*(p_C,p_N,p_P)}{\partial p_C}=R^*;\frac{\partial\pi^*(p_C,p_N,p_P)}{\partial p_N}=-N^*;\frac{\partial\pi^*(p_C,p_N,p_P)}{\partial p_P}=-P^*.$$

Proposition 10

After dividing the first postulate of (34) by the second and then the first by the third, the following is obtained:

$$\frac{\partial p_N}{\partial p_C}\approx-\frac{R^*}{N^*};\frac{\partial p_P}{\partial p_C}\approx-\frac{R^*}{P^*}$$

35)

Nevertheless, it may happen that, despite changes in the prices of outputs and fertilizers, it is better, in terms of profits, that the previous fertilization OED is still used or even that it is increased. This would happen if the positive changes produced by an increase in the price of the output outweighed the decrease in profits due to the increase in costs due to the rise in fertilizer prices. This event is also explained by the previous analysis.

In most cases, the optimal-economic doses of fertilization applied in Mexican agriculture are invalid, inefficient, and anachronistic because environmental and ecological costs are not considered and

DOI: https://doi.org/10.29312/remexca.v16i4.3425

they are considered fixed and immutable, so they are erroneously not revised and updated when the input-output price ratios change.

Unfortunately, for ease, the optimal-economic dose for a crop and for a specific location is calculated directly from the experimental data, without prior estimation of the response surfaces. This is why frequently revising OED and updating them for 100 crops and 70 regions would be very costly and inefficient. In addition, the environmental costs that occur with chemical fertilization in agriculture must also be considered in the estimation of response surfaces (González-Estrada and Camacho, 2018, 2017).

Without considering these costs, fertilization OED could not be economically optimal or efficient from the point of view of social and environmental wellbeing. In the document that considers the implementation of mitigation actions to meet the emission reduction objectives committed to in the Paris Agreement by Mexico (INECC, 2021), actions for the control of greenhouse gas emissions produced by nitrogen chemical fertilization are not considered despite the fact that this is the main source of emissions from Mexican agriculture. For these reasons, a new science and technology policy for the Mexican countryside is urgent (González-Estrada, 2020).

Conclusions

The ten propositions obtained validate the central hypothesis of this research. Not updating the optimal-economic fertilization doses when the prices of outputs or fertilizers change so significantly often produces inefficiency, reduced farmers' profits, and higher greenhouse gas emissions. The use of chemical fertilizers in agriculture is inefficient and irrational because fertilization OEDs are not followed and because the costs of emissions produced by nitrogen fertilizers are not considered.

Fertilizer use efficiency and application practices should be improved; the use of slow-release nitrogen fertilizers, the application of organic fertilizers, and the use of microorganisms that improve plant nutrition should be promoted. It is recommended to estimate the response surfaces for each crop and for each of the 72 agricultural regions of Mexico. The calculation of the optimal-economic fertilization doses must also consider the environmental costs produced by nitrogen chemical fertilization in agriculture.

Likewise, it is proposed to create a database with the experimental results of fertilization for each crop and for each of the agricultural regions of the country. It is also proposed to develop a computerized system that updates the optimal-economic doses of fertilization immediately after the new price vectors are known, for each crop and for each of the regions of the country. Finally, it is also suggested that the efficient control of greenhouse gas emissions produced by nitrogen chemical fertilization in Mexican agriculture be included in the implementation of mitigation actions to meet the emission reduction objectives committed to in the Paris Agreement by Mexico.

Fertilization OED with environmental costs will reduce expected yields, so there is an urgent need for a new science and technology policy for the countryside that is oriented towards sustainable development, that improves yields and profitability, and that adapts to the emission control required to avoid the catastrophic effects of climate change.

Bibliography

1 Borlaug, N. and Doesell, Ch. 2004. Prospects for world agriculture in the twenty-first century, Ed. Sustainable Agriculture and the International Rice-Wheat System. Marcel Dekker, Inc. New York. 5-22 pp.

2 Chiang, A. and Wainwright K. 2012. Fundamental methods of mathematical economics. Fourth Ed. McGraw-Hill Companies. New York. 708 p.

3 González-Estrada, A. 1990. Los tipos de agricultura y las regiones agrícolas de México. Colegio de Posgraduados. Chapingo, Estado de México. 152 p.

Revista Mexicana de Ciencias Agrícolas
REMEXCA
DOI: https://doi.org/10.29312/remexca.v16i4.3425

4 González-Estrada, A. 1995. Algoritmo para actualizar las dosis óptimo-económicas de fertilización al cambiar la relación de precios. Agricultura Técnica en México. 11(1):69-86.

5 González-Estrada, A. 2020. Hacia una nueva política científica y tecnológica para el desarrollo del campo mexicano. Ciencia, Tecnología e innovación. *In*: ciencia, tecnología e innovación: una visión desde el poder legislativo. Editorial Fontamara y Cámara de Diputados. Ciudad de México. 225-240 pp.

6 González-Estrada, A. 2022. Programación matemática no-lineal con aplicaciones en la economía. División de Ciencias Económico-Administrativas, Universidad Autónoma Chapingo. Chapingo, Estado de México. 221 p.

7 González-Estrada, A. y Camacho, M. A. 2017. Emisiones de gases de efecto invernadero de la fertilización nitrogenada en México. Revista Mexicana de Ciencias Agrícolas. 8(8):1733-1745.

8 González-Estrada, A. y CamachoA. 2018. Costos y políticas eficientes de control de emisiones de la fertilización nitrogenada en la agricultura mexicana. Revista Mexicana de Ciencias Agrícolas. 9(7):1399-1410.

9 Heffer, P. and Prud'homme, M. 2008. Outlook for world fertilizer demand, supply, and supply/ demand balance. Turkish Journal of Agriculture and Forestry and International Fertilizer Association. 32(28):159-174.

10 INECC. 2021. Instituto Nacional de Ecología y Cambio Climático. Estimación de costos y beneficios asociados a la implementación de acciones de mitigación para el cumplimiento de los objetivos de reducción de emisiones comprometidos en el Acuerdo de París. Ciudad de México. 202 p.

11 IFA. 2018. International Fertilizer Association. Estimating and reporting fertilizer-related greenhouse gas emissions. A discussion paper for policy makers. 11 p.

12 Jehle, G. A. and Reny, P. J. 2011. Advanced microeconomic theory. Prentice Hall. New York. 656 p.

13 Mas-Colell, A.; Whinston, M. D. and Green, J. R. 1995. Microeconomic theory. Oxford University Press. Cambridge, England. 1008 p.

14 Simon, C. P. and Blume, L. 1994. Mathematics for economists. WW. Norton & Co. New York. 930 p.

15 Siwa, M. and Rosegrant, M. W. 2015. Energy and agriculture: evolving dynamics and Future implications. *In*: sustainable economic development, resources, environment, and institutions: 261-292. Academic Press, Elsevier. San Diego, CA. 532 p.

16 Sydsaeter, K.; Hammond, P.; Seierstad, A. and Strom A. 2008. Further mathematics for economic analysis. Second edition. Prentice Hall. London. 616 p.

17 Takayama, A. 1991. Mathematical economics. Cambridge University Press. New York. 737 p.

18 Takayama, A. 1996. Analytical methods in economics. The University of Michigan Press. Ann Arbor, Michigan. 672 p.

19 Varian, H. R. 1992. Microeconomic analysis. WW. Norton & Company. New York. 591 p.

20 World-Bank. 2022a. Commodity markets outlook. Washington, DC. 48 p.

21 World-Bank. 2022b. Commodity prices forecasts. Released Washington, DC. 2 p.

Adjustment to optimal-economic fertilization doses when prices change: economic-mathematical analysis

Journal Information
Journal ID (publisher-id): remexca
Title: Revista mexicana de ciencias agrícolas
Abbreviated Title: Rev. Mex. Cienc. Agríc
ISSN (print): 2007-0934
Publisher: Instituto Nacional de Investigaciones Forestales, Agrícolas y Pecuarias

Article/Issue Information
Date received: 01 February 2025
Date accepted: 01 June 2025
Publication date: 27 June 2025
Publication date: May-Jun 2025
Volume: 16
Issue: 4
Electronic Location Identifier: e3425
DOI: 10.29312/remexca.v16i4.3425

Categories

Subject: Articles

Keywords:

Keywords:

analysis behind the envelope

fertilization dose

input-output price ratio

Counts

Figures: 1

Tables: 0

Equations: 29

References: 21

Pages: 0