Article

Design of 2^n factorial experiments for their analysis with Infostat and Infogen

Delfina de Jesús Pérez López
J. Ramón Pascual Franco Martínez
Francisco Gutiérrez Rodríguez
Jesús Hernández Ávila
Artemio Balbuena Melgarejo
Andrés González Huerta[§]

Center for Research and Advanced Studies in Plant Breeding-Faculty of Agricultural Sciences-Autonomous University of the State of Mexico. 'El Cerrillo' University *Campus*, Toluca, State of Mexico, Mexico. AP. 435. Tel. 7222965518, ext. 148. (djperezl@uaemex.mx; jrfrancom@uaemex.mx; fgutierrezr@uaemex.mx; jhernandeza@uaemex.mx; abalbuenam@uaemex.mx).

[§]Corresponding author: agonzalezh@uaemex.mx.

Abstract

The design and analysis of data from factorial experiments using statistical packages saves time, simplifies calculations, and allows discussion of the individual or joint contribution of several factors. This study indicates the procedures for analyzing data with the InfoStat and InfoGen statistical packages in 2^n factorial experiments, with n= 2, 3, 4, 5, 6, 7 and 8 factors of productivity. In addition, the design of its treatment structure, possible interactions, statistical models for a randomize complete block design and some formulas for calculating sums of squares for n= 8 are shown. The relationships between the linear models are established and also the procedures that must be applied in both statistical packages to obtain an analysis of variance and the comparison of treatment means if the general statistical model is used, the instructions to estimate the effects for each of the eight main factors and for their possible interactions with the Tukey test (p= 0.01) are also indicated. The design of this type of experiments was made to be analyzed with academic test versions, which are free for six months in both statistical packages. The user could purchase a license for one year to install it on two personal computers paying, for each software, only USD$50.00. Both statistical packages are very friendly and can be used to analyze any experimental design if the appropriate statistical model is indicated.

Keywords: construction of factorial treatments, exploratory experimental design, randomized complete blocks, statistical models.

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Introduction

The sequential and iterative nature of the response surface methodology requires that in each phase significant factors be identified, model adaptation is made, and the direction in which the optimal experimental conditions are determined. To identify the factors that have the greatest influence on the response and to examine the goodness of fit of the statistical model, the analysis of variance should be used (Jiménez, 2015).

In this context, the design and analysis of factorial experiments in randomized complete blocks has been used as a very valuable tool in agricultural sciences, and in other branches of science and technology (Sahagún, 1998; Medina and López, 2011; Jiménez, 2015; Álvarez *et al*., 2018), in addition to determining the contribution of each of the factors studied, it allows simultaneously knowing what are the effects of their interactions, in each trial and through years, localities or their combinations in time and/or space (Montgomery, 2010; Gomez and Gomez, 1984; Di Rienzo *et al*., 2008; Balzarini and Di Rienzo, 2016).

The 2^n experiments are families of tests with n factors and two levels within each of them that generally represent very wide intervals of variability in exploratory studies, when there is no or when there is little published information and consequently, it is desirable to redesign their research with a more important subset (Gomez and Gomez, 1984; Montgomery, 2010; Medina and López, 2011; Jiménez, 2015).

The evaluation of two promising crosses formed with the most outstanding lines, the introduction to a new region of cash crops, of plant species in the process of domestication or of animals of various breeds, as well as the generation, validation, application and transfer of technology could represent a desirable case for the design and analysis of this type of research (Torres *et al*., 2011; Reynoso *et al*., 2014; Torres *et al*., 2017; González *et al*., 2019). The determination of optimal doses when organic and inorganic fertilizers are applied, in different population densities, on several planting dates, or different localities and years, could have as a prerequisite the analysis of data from exploratory trials (Rodríguez *et al*., 2015; Quiroz *et al*., 2017; Padilla *et al*., 2019; González *et al*., 2019).

The 2^n factorial experiments are difficult to analyze with a desktop calculator when n increases, due to an increase in the number of interactions to be estimated, when n= 6, 7 and 8, there are 57, 120 and 247 interactions, respectively. With the use of statistical packages, their design and analysis save time and simplify calculations, especially in series of experiments or in arrangements of experimental units that are designed to facilitate their handling in the field and to obtain different levels of precision in the estimation of the factors that make them up (Gomez and Gomez, 1984; Martínez, 1988; Padilla *et al*., 2019; González *et al*., 2019).

The first versions of InfoStat and InfoGen appeared before 2008 and 2016, respectively, both were created by programmers and analysts from the University of Córdoba, in Argentina and their academic versions can be downloaded for free from their websites.

Both programs are very friendly and can be used to analyze any type of experiment if the correct statistical model is introduced, because these can now enhance its use; through R-software (Sahagún, 1998; Balzarini *et al*., 2008; Di Rienzo *et al*., 2008; Balzarini and Di Rienzo, 2016). As a reference in the use of the SAS (Statistical Analysis System, 1989); InfoStat and InfoGen; Padilla *et al*. (2019); González *et al*. (2019) analyzed data on the production in green pod of three broad bean cultivars (*Vicia faba* L.), planted in two Mexican localities, with three population densities and three fertilization formulas: in the first publication, the codes to analyze and validate a series of experiments in randomized complete blocks in arrangement of subdivided plots were presented and in the second publication, the program to estimate mutually orthogonal contrasts in this type of trials was described.

In the above context, the main objective of this study was to design the procedures for analyzing 2^n factorial experiments in randomized complete blocks with r repetitions using InfoStat and InfoGen, to generate an analysis of variance and a comparison of treatment means with the Tukey test.

Materials and methods

Notation used

The eight factors will be represented as A, B, H; both levels within each of these will be designated by the same letter, in lowercase and with a subscript, as a_1 and a_2 for the factor A. Interactions will be designated as A∗B, A∗C, A∗B∗C∗D∗E∗F∗G∗H. In statistical models, each factor will be associated with a subscript: A, B, C, D, E, F, G, H will correspond to i, j, k, l, m, n, o, p, respectively. The repetitions will be identified with R and the last subscript that is indicated in the residual of each model used (Sahagún, 1998; Balzarini *et al*., 2008; Balzarini and Di Rienzo, 2016).

Number of treatments and their factorial structure

Treatments will be represented by the combination of levels of two or more factors. Experimental design is a strategy of combining the structure of treatments with the structure of experimental units in such a way that alterations in responses, of at least in some subset of experimental units, can be attributed exclusively to the action of treatments, except for random variations. Thus, it is possible to compare treatment means or linear combinations of treatment means with as little noise as possible (Di Rienzo *et al*., 2008; Balzarini *et al*., 2016; Balzarini and Di Rienzo, 2016). Table 1 shows the terminology, how many and what is the structure of treatments for n= 2, 3, 4, 5, 6, 7, 8.

Table 1. Number of factors, codes to represent them, number and structure of treatments.

n	Symbology	Number	Treatments that will be analyzed
2	A, B	4	a_1b_1, a_1b_2, a_2b_1, a_2b_2
3	A, B, C	8	$a_1b_1c_1$, $a_1b_1c_2$, $a_1b_2c_1$, $a_1b_2c_2$, $a_2b_1c_1$, $a_2b_1c_2$, $a_2b_2c_1$, $a_2b_2c_2$

n	Symbology	Number	Treatments that will be analyzed
4	A, B, C, D	16	$a_1b_1c_1d_1$, $a_1b_1c_1d_2$, $a_1b_1c_2d_1$, $a_1b_1c_2d_2$, $a_1b_2c_1d_1$, $a_1b_2c_1d_2$, $a_1b_2c_2d_1$, $a_1b_2c_2d_2$, $a_2b_1c_1d_1$, $a_2b_1c_1d_2$, $a_2b_1c_2d_1$, $a_2b_1c_2d_2$, $a_2b_2c_1d_1$, $a_2b_2c_1d_2$, $a_2b_2c_2d_1$, $a_2b_2c_2d_2$
5	A, B, C, D, E	32	$a_1b_1c_1d_1e_1$, $a_1b_1c_1d_1e_2$, $a_1b_1c_1d_2e_1$, $a_1b_1c_1d_2e_2$, $a_1b_1c_2d_1e_1$, $a_1b_1c_2d_1e_2$, $a_1b_1c_2d_2e_1$, $a_1b_1c_2d_2e_2$, $a_1b_2c_1d_1e_1$, $a_1b_2c_1d_1e_2$, $a_1b_2c_1d_2e_1$, $a_1b_2c_1d_2e_2$, $a_1b_2c_2d_1e_1$, $a_1b_2c_2d_1e_2$, $a_1b_2c_2d_2e_1$, $a_1b_2c_2d_2e_2$, $a_2b_1c_1d_1e_1$, $a_2b_1c_1d_1e_2$, $a_2b_1c_1d_2e_1$, $a_2b_1c_1d_2e_2$, $a_2b_1c_2d_1e_1$, $a_2b_1c_2d_1e_2$, $a_2b_1c_2d_2e_1$, $a_2b_1c_2d_2e_2$, $a_2b_2c_1d_1e_1$, $a_2b_2c_1d_1e_2$, $a_2b_2c_1d_2e_1$, $a_2b_2c_1d_2e_2$, $a_2b_2c_2d_1e_1$, $a_2b_2c_2d_1e_2$, $a_2b_2c_2d_2e_1$, $a_2b_2c_2d_2e_2$
6	A, B, C, D, E, F	64	$a_1b_1c_1d_1e_1f_1$, $a_1b_1c_1d_1e_1f_2$, $a_1b_1c_1d_1e_2f_1$, $a_1b_1c_1d_1e_2f_2$, $a_1b_1c_1d_2e_1f_1$, $a_1b_1c_1d_2e_1f_2$, $a_1b_1c_1d_2e_2f_1$, $a_1b_1c_1d_2e_2f_2$, $a_1b_1c_2d_1e_1f_1$, $a_1b_1c_2d_1e_1f_2$, $a_1b_1c_2d_1e_2f_1$, $a_1b_1c_2d_1e_2f_2$, $a_1b_1c_2d_2e_1f_1$, $a_1b_1c_2d_2e_1f_2$, $a_1b_1c_2d_2e_2f_1$, $a_1b_1c_2d_2e_2f_2$, $a_1b_2c_1d_1e_1f_1$, $a_1b_2c_1d_1e_1f_2$, $a_1b_2c_1d_1e_2f_1$, $a_1b_2c_1d_1e_2f_2$, $a_1b_2c_1d_2e_1f_1$, $a_1b_2c_1d_2e_1f_2$, $a_1b_2c_1d_2e_2f_1$, $a_1b_2c_1d_2e_2f_2$, $a_1b_2c_2d_1e_1f_1$, $a_1b_2c_2d_1e_1f_2$, $a_1b_2c_2d_1e_2f_1$, $a_1b_2c_2d_1e_2f_2$, $a_1b_2c_2d_2e_1f_1$, $a_1b_2c_2d_2e_1f_2$, $a_1b_2c_2d_2e_2f_1$, $a_1b_2c_2d_2e_2f_2$, $a_2b_1c_1d_1e_1f_1$, $a_2b_1c_1d_1e_1f_2$, $a_2b_1c_1d_1e_2f_1$, $a_2b_1c_1d_1e_2f_2$, $a_2b_1c_1d_2e_1f_1$, $a_2b_1c_1d_2e_1f_2$, $a_2b_1c_1d_2e_2f_1$, $a_2b_1c_1d_2e_2f_2$, $a_2b_1c_2d_1e_1f_1$, $a_2b_1c_2d_1e_1f_2$, $a_2b_1c_2d_1e_2f_1$, $a_2b_1c_2d_1e_2f_2$, $a_2b_1c_2d_2e_1f_1$, $a_2b_1c_2d_2e_1f_2$, $a_2b_1c_2d_2e_2f_1$, $a_2b_1c_2d_2e_2f_2$, $a_2b_2c_1d_1e_1f_1$, $a_2b_2c_1d_1e_1f_2$, $a_2b_2c_1d_1e_2f_1$, $a_2b_2c_1d_1e_2f_2$, $a_2b_2c_1d_2e_1f_1$, $a_2b_2c_1d_2e_1f_2$, $a_2b_2c_1d_2e_2f_1$, $a_2b_2c_1d_2e_2f_2$, $a_2b_2c_2d_1e_1f_1$, $a_2b_2c_2d_1e_1f_2$, $a_2b_2c_2d_1e_2f_1$, $a_2b_2c_2d_1e_2f_2$, $a_2b_2c_2d_2e_1f_1$, $a_2b_2c_2d_2e_1f_2$, $a_2b_2c_2d_2e_2f_1$, $a_2b_2c_2d_2e_2f_2$
7	A, B, C, D, E, F, G	128	$a_1b_1c_1d_1e_1f_1g_1$, $a_1b_1c_1d_1e_1f_1g_2$, $a_1b_1c_1d_1e_1f_2g_1$, $a_1b_1c_1d_1e_1f_2g_2$, $a_1b_1c_1d_1e_2f_1g_1$, $a_1b_1c_1d_1e_2f_1g_2$, $a_1b_1c_1d_1e_2f_2g_1$, $a_1b_1c_1d_1e_2f_2g_2$, $a_1b_1c_1d_2e_1f_1g_1$, $a_1b_1c_1d_2e_1f_1g_2$, $a_1b_1c_1d_2e_1f_2g_1$, $a_1b_1c_1d_2e_1f_2g_2$, $a_1b_1c_1d_2e_2f_1g_1$, $a_1b_1c_1d_2e_2f_1g_2$, $a_1b_1c_1d_2e_2f_2g_1$, $a_1b_1c_1d_2e_2f_2g_2$, $a_1b_1c_2d_1e_1f_1g_1$, $a_1b_1c_2d_1e_1f_1g_2$, $a_1b_1c_2d_1e_1f_2g_1$, $a_1b_1c_2d_1e_1f_2g_2$, $a_1b_1c_2d_1e_2f_1g_1$, $a_1b_1c_2d_1e_2f_1g_2$, $a_1b_1c_2d_1e_2f_2g_1$, $a_1b_1c_2d_1e_2f_2g_2$, $a_1b_1c_2d_2e_1f_1g_1$, $a_1b_1c_2d_2e_1f_1g_2$, $a_1b_1c_2d_2e_1f_2g_1$, $a_1b_1c_2d_2e_1f_2g_2$, $a_1b_1c_2d_2e_2f_1g_1$, $a_1b_1c_2d_2e_2f_1g_2$, $a_1b_1c_2d_2e_2f_2g_1$, $a_1b_1c_2d_2e_2f_2g_2$,

n	Symbology	Number	Treatments that will be analyzed
			$a_1b_2c_1d_1e_1f_1g_1$, $a_1b_2c_1d_1e_1f_1g_2$, $a_1b_2c_1d_1e_1f_2g_1$, $a_1b_2c_1d_1e_1f_2g_2$, $a_1b_2c_1d_1e_2f_1g_1$, $a_1b_2c_1d_1e_2f_1g_2$, $a_1b_2c_1d_1e_2f_2g_1$, $a_1b_2c_1d_1e_2f_2g_2$, $a_1b_2c_1d_2e_1f_1g_1$, $a_1b_2c_1d_2e_1f_1g_2$, $a_1b_2c_1d_2e_1f_2g_1$, $a_1b_2c_1d_2e_1f_2g_2$, $a_1b_2c_1d_2e_2f_1g_1$, $a_1b_2c_1d_2e_2f_1g_2$, $a_1b_2c_1d_2e_2f_2g_1$, $a_1b_2c_1d_2e_2f_2g_2$, $a_1b_2c_2d_1e_1f_1g_1$, $a_1b_2c_2d_1e_1f_1g_2$, $a_1b_2c_2d_1e_1f_2g_1$, $a_1b_2c_2d_1e_1f_2g_2$, $a_1b_2c_2d_1e_2f_1g_1$, $a_1b_2c_2d_1e_2f_1g_2$, $a_1b_2c_2d_1e_2f_2g_1$, $a_1b_2c_2d_1e_2f_2g_2$, $a_1b_2c_2d_2e_1f_1g_1$, $a_1b_2c_2d_2e_1f_1g_2$, $a_1b_2c_2d_2e_1f_2g_1$, $a_1b_2c_2d_2e_1f_2g_2$, $a_1b_2c_2d_2e_2f_1g_1$, $a_1b_2c_2d_2e_2f_1g_2$, $a_1b_2c_2d_2e_2f_2g_1$, $a_1b_2c_2d_2e_2f_2g_2$, $a_2b_1c_1d_1e_1f_1g_1$, $a_2b_1c_1d_1e_1f_1g_2$, $a_2b_1c_1d_1e_1f_2g_1$, $a_2b_1c_1d_1e_1f_2g_2$, $a_2b_1c_1d_1e_2f_1g_1$, $a_2b_1c_1d_1e_2f_1g_2$, $a_2b_1c_1d_1e_2f_2g_1$, $a_2b_1c_1d_1e_2f_2g_2$, $a_2b_1c_1d_2e_1f_1g_1$, $a_2b_1c_1d_2e_1f_1g_2$, $a_2b_1c_1d_2e_1f_2g_1$, $a_2b_1c_1d_2e_1f_2g_2$, $a_2b_1c_1d_2e_2f_1g_1$, $a_2b_1c_1d_2e_2f_1g_2$, $a_2b_1c_1d_2e_2f_2g_1$, $a_2b_1c_1d_2e_2f_2g_2$, $a_2b_1c_2d_1e_1f_1g_1$, $a_2b_1c_2d_1e_1f_1g_2$, $a_2b_1c_2d_1e_1f_2g_1$, $a_2b_1c_2d_1e_1f_2g_2$, $a_2b_1c_2d_1e_2f_1g_1$, $a_2b_1c_2d_1e_2f_1g_2$, $a_2b_1c_2d_1e_2f_2g_1$, $a_2b_1c_2d_1e_2f_2g_2$, $a_2b_1c_2d_2e_1f_1g_1$, $a_2b_1c_2d_2e_1f_1g_2$, $a_2b_1c_2d_2e_1f_2g_1$, $a_2b_1c_2d_2e_1f_2g_2$, $a_2b_1c_2d_2e_2f_1g_1$, $a_2b_1c_2d_2e_2f_1g_2$, $a_2b_1c_2d_2e_2f_2g_1$, $a_2b_1c_2d_2e_2f_2g_2$, $a_2b_2c_1d_1e_1f_1g_1$, $a_2b_2c_1d_1e_1f_1g_2$, $a_2b_2c_1d_1e_1f_2g_1$, $a_2b_2c_1d_1e_1f_2g_2$, $a_2b_2c_1d_1e_2f_1g_1$, $a_2b_2c_1d_1e_2f_1g_2$, $a_2b_2c_1d_1e_2f_2g_1$, $a_2b_2c_1d_1e_2f_2g_2$, $a_2b_2c_1d_2e_1f_1g_1$, $a_2b_2c_1d_2e_1f_1g_2$, $a_2b_2c_1d_2e_1f_2g_1$, $a_2b_2c_1d_2e_1f_2g_2$, $a_2b_2c_1d_2e_2f_1g_1$, $a_2b_2c_1d_2e_2f_1g_2$, $a_2b_2c_1d_2e_2f_2g_1$, $a_2b_2c_1d_2e_2f_2g_2$, $a_2b_2c_2d_1e_1f_1g_1$, $a_2b_2c_2d_1e_1f_1g_2$, $a_2b_2c_2d_1e_1f_2g_1$, $a_2b_2c_2d_1e_1f_2g_2$, $a_2b_2c_2d_1e_2f_1g_1$, $a_2b_2c_2d_1e_2f_1g_2$, $a_2b_2c_2d_1e_2f_2g_1$, $a_2b_2c_2d_1e_2f_2g_2$, $a_2b_2c_2d_2e_1f_1g_1$, $a_2b_2c_2d_2e_1f_1g_2$, $a_2b_2c_2d_2e_1f_2g_1$, $a_2b_2c_2d_2e_1f_2g_2$, $a_2b_2c_2d_2e_2f_1g_1$, $a_2b_2c_2d_2e_2f_1g_2$, $a_2b_2c_2d_2e_2f_2g_1$, $a_2b_2c_2d_2e_2f_2g_2$
8	A, B, C, D, E, F, G, H	256	$a_1b_1c_1d_1e_1f_1g_1h_1$, $a_1b_1c_1d_1e_1f_1g_1h_2$, $a_1b_1c_1d_1e_1f_1g_2h_1$, $a_1b_1c_1d_1e_1f_1g_2h_2$, $a_1b_1c_1d_1e_1f_2g_1h_1$, $a_1b_1c_1d_1e_1f_2g_1h_2$, $a_1b_1c_1d_1e_1f_2g_2h_1$, $a_1b_1c_1d_1e_1f_2g_2h_2$, $a_1b_1c_1d_1e_2f_1g_1h_1$,$a_1b_1c_1d_1e_2f_1g_1h_2$, $a_1b_1c_1d_1e_2f_1g_2h_1$, $a_1b_1c_1d_1e_2f_1g_2h_2$, $a_1b_1c_1d_1e_2f_2g_1h_1$, $a_1b_1c_1d_1e_2f_2g_1h_2$, $a_1b_1c_1d_1e_2f_2g_2h_1$,$a_1b_1c_1d_1e_2f_2g_2h_2$, $a_1b_1c_1d_2e_1f_1g_1h_1$, $a_1b_1c_1d_2e_1f_1g_1h_2$,$a_1b_1c_1d_2e_1f_1g_2h_1$, $a_1b_1c_1d_2e_1f_1g_2h_2$, $a_1b_1c_1d_2e_1f_2g_1h_1$,$a_1b_1c_1d_2e_1f_2g_1h_2$, $a_1b_1c_1d_2e_1f_2g_2h_1$, $a_1b_1c_1d_2e_1f_2g_2h_2$, $a_1b_1c_1d_2e_2f_1g_1h_1$, $a_1b_1c_1d_2e_2f_1g_1h_2$, $a_1b_1c_1d_2e_2f_1g_2h_1$,$a_1b_1c_1d_2e_2f_1g_2h_2$, $a_1b_1c_1d_2e_2f_2g_1h_1$, $a_1b_1c_1d_2e_2f_2g_1h_2$,$a_1b_1c_1d_2e_2f_2g_2h_1$, $a_1b_1c_1d_2e_2f_2g_2h_2$, $a_1b_1c_2d_1e_1f_1g_1h_1$,$a_1b_1c_2d_1e_1f_1g_1h_2$, $a_1b_1c_2d_1e_1f_1g_2h_1$, $a_1b_1c_2d_1e_1f_1g_2h_2$, $a_1b_1c_2d_1e_1f_2g_1h_1$, $a_1b_1c_2d_1e_1f_2g_1h_2$, $a_1b_1c_2d_1e_1f_2g_2h_1$,$a_1b_1c_2d_1e_1f_2g_2h_2$, $a_1b_1c_2d_1e_2f_1g_1h_1$, $a_1b_1c_2d_1e_2f_1g_1h_2$, $a_1b_1c_2d_1e_2f_1g_2h_1$, $a_1b_1c_2d_1e_2f_1g_2h_2$, $a_1b_1c_2d_1e_2f_2g_1h_1$,$a_1b_1c_2d_1e_2f_2g_1h_2$, $a_1b_1c_2d_1e_2f_2g_2h_1$, $a_1b_1c_2d_1e_2f_2g_2h_2$,

n	Symbology	Number	Treatments that will be analyzed
			$a_1b_1c_2d_2e_1f_1g_1h_1$, $a_1b_1c_2d_2e_1f_1g_1h_2$, $a_1b_1c_2d_2e_1f_1g_2h_1$,
			$a_1b_1c_2d_2e_1f_1g_2h_2$, $a_1b_1c_2d_2e_1f_2g_1h_1$, $a_1b_1c_2d_2e_1f_2g_1h_2$,
			$a_1b_1c_2d_2e_1f_2g_2h_1$, $a_1b_1c_2d_2e_1f_2g_2h_2$, $a_1b_1c_2d_2e_2f_1g_1h_1$,
			$a_1b_1c_2d_2e_2f_1g_1h_2$, $a_1b_1c_2d_2e_2f_1g_2h_1$, $a_1b_1c_2d_2e_2f_1g_2h_2$,
			$a_1b_1c_2d_2e_2f_2g_1h_1$, $a_1b_1c_2d_2e_2f_2g_1h_2$, $a_1b_1c_2d_2e_2f_2g_2h_1$,
			$a_1b_1c_2d_2e_2f_2g_2h_2$, $a_1b_2c_1d_1e_1f_1g_1h_1$, $a_1b_2c_1d_1e_1f_1g_1h_2$,
			$a_1b_2c_1d_1e_1f_1g_2h_1$, $a_1b_2c_1d_1e_1f_1g_2h_2$, $a_1b_2c_1d_1e_1f_2g_1h_1$,
			$a_1b_2c_1d_1e_1f_2g_1h_2$, $a_1b_2c_1d_1e_1f_2g_2h_1$, $a_1b_2c_1d_1e_1f_2g_2h_2$,
			$a_1b_2c_1d_1e_2f_1g_1h_1$, $a_1b_2c_1d_1e_2f_1g_1h_2$, $a_1b_2c_1d_1e_2f_1g_2h_1$,
			$a_1b_2c_1d_1e_2f_1g_2h_2$, $a_1b_2c_1d_1e_2f_2g_1h_1$, $a_1b_2c_1d_1e_2f_2g_1h_2$,
			$a_1b_2c_1d_1e_2f_2g_2h_1$, $a_1b_2c_1d_1e_2f_2g_2h_2$, $a_1b_2c_1d_2e_1f_1g_1h_1$,
			$a_1b_2c_1d_2e_1f_1g_1h_2$, $a_1b_2c_1d_2e_1f_1g_2h_1$, $a_1b_2c_1d_2e_1f_1g_2h_2$,
			$a_1b_2c_1d_2e_1f_2g_1h_1$, $a_1b_2c_1d_2e_1f_2g_1h_2$, $a_1b_2c_1d_2e_1f_2g_2h_1$,
			$a_1b_2c_1d_2e_1f_2g_2h_2$, $a_1b_2c_1d_2e_2f_1g_1h_1$, $a_1b_2c_1d_2e_2f_1g_1h_2$,
			$a_1b_2c_1d_2e_2f_1g_2h_1$, $a_1b_2c_1d_2e_2f_1g_2h_2$, $a_1b_2c_1d_2e_2f_2g_1h_1$,
			$a_1b_2c_1d_2e_2f_2g_1h_2$, $a_1b_2c_1d_2e_2f_2g_2h_1$, $a_1b_2c_1d_2e_2f_2g_2h_2$,
			$a_1b_2c_2d_1e_1f_1g_1h_1$, $a_1b_2c_2d_1e_1f_1g_1h_2$, $a_1b_2c_2d_1e_1f_1g_2h_1$,
			$a_1b_2c_2d_1e_1f_1g_2h_2$, $a_1b_2c_2d_1e_1f_2g_1h_1$, $a_1b_2c_2d_1e_1f_2g_1h_2$,
			$a_1b_2c_2d_1e_1f_2g_2h_1$, $a_1b_2c_2d_1e_1f_2g_2h_2$, $a_1b_2c_2d_1e_2f_1g_1h_1$,
			$a_1b_2c_2d_1e_2f_1g_1h_2$, $a_1b_2c_2d_1e_2f_1g_2h_1$, $a_1b_2c_2d_1e_2f_1g_2h_2$,
			$a_1b_2c_2d_1e_2f_2g_1h_1$, $a_1b_2c_2d_1e_2f_2g_1h_2$, $a_1b_2c_2d_1e_2f_2g_2h_1$,
			$a_1b_2c_2d_1e_2f_2g_2h_2$, $a_1b_2c_2d_2e_1f_1g_1h_1$, $a_1b_2c_2d_2e_1f_1g_1h_2$,
			$a_1b_2c_2d_2e_1f_1g_2h_1$, $a_1b_2c_2d_2e_1f_1g_2h_2$, $a_1b_2c_2d_2e_1f_2g_1h_1$,
			$a_1b_2c_2d_2e_1f_2g_1h_2$, $a_1b_2c_2d_2e_1f_2g_2h_1$, $a_1b_2c_2d_2e_1f_2g_2h_2$,
			$a_1b_2c_2d_2e_2f_1g_1h_1$, $a_1b_2c_2d_2e_2f_1g_1h_2$, $a_1b_2c_2d_2e_2f_1g_2h_1$,
			$a_1b_2c_2d_2e_2f_1g_2h_2$, $a_1b_2c_2d_2e_2f_2g_1h_1$, $a_1b_2c_2d_2e_2f_2g_1h_2$,
			$a_1b_2c_2d_2e_2f_2g_2h_1$, $a_1b_2c_2d_2e_2f_2g_2h_2$, $a_2b_1c_1d_1e_1f_1g_1h_1$,
			$a_2b_1c_1d_1e_1f_1g_1h_2$, $a_2b_1c_1d_1e_1f_1g_2h_1$, $a_2b_1c_1d_1e_1f_1g_2h_2$,
			$a_2b_1c_1d_1e_1f_2g_1h_1$, $a_2b_1c_1d_1e_1f_2g_1h_2$, $a_2b_1c_1d_1e_1f_2g_2h_1$,
			$a_2b_1c_1d_1e_1f_2g_2h_2$, $a_2b_1c_1d_1e_2f_1g_1h_1$, $a_2b_1c_1d_1e_2f_1g_1h_2$,
			$a_2b_1c_1d_1e_2f_1g_2h_1$, $a_2b_1c_1d_1e_2f_1g_2h_2$, $a_2b_1c_1d_1e_2f_2g_1h_1$,
			$a_2b_1c_1d_1e_2f_2g_1h_2$, $a_2b_1c_1d_1e_2f_2g_2h_1$, $a_2b_1c_1d_1e_2f_2g_2h_2$,
			$a_2b_1c_1d_2e_1f_1g_1h_1$, $a_2b_1c_1d_2e_1f_1g_1h_2$, $a_2b_1c_1d_2e_1f_1g_2h_1$,
			$a_2b_1c_1d_2e_1f_1g_2h_2$, $a_2b_1c_1d_2e_1f_2g_1h_1$, $a_2b_1c_1d_2e_1f_2g_1h_2$,
			$a_2b_1c_1d_2e_1f_2g_2h_1$, $a_2b_1c_1d_2e_1f_2g_2h_2$, $a_2b_1c_1d_2e_2f_1g_1h_1$,
			$a_2b_1c_1d_2e_2f_1g_1h_2$, $a_2b_1c_1d_2e_2f_1g_2h_1$, $a_2b_1c_1d_2e_2f_1g_2h_2$,
			$a_2b_1c_1d_2e_2f_2g_1h_1$, $a_2b_1c_1d_2e_2f_2g_1h_2$, $a_2b_1c_1d_2e_2f_2g_2h_1$,
			$a_2b_1c_1d_2e_2f_2g_2h_2$, $a_2b_1c_2d_1e_1f_1g_1h_1$, $a_2b_1c_2d_1e_1f_1g_1h_2$,
			$a_2b_1c_2d_1e_1f_1g_2h_1$, $a_2b_1c_2d_1e_1f_1g_2h_2$, $a_2b_1c_2d_1e_1f_2g_1h_1$,
			$a_2b_1c_2d_1e_1f_2g_1h_2$, $a_2b_1c_2d_1e_1f_2g_2h_1$, $a_2b_1c_2d_1e_1f_2g_2h_2$,
			$a_2b_1c_2d_1e_2f_1g_1h_1$, $a_2b_1c_2d_1e_2f_1g_1h_2$, $a_2b_1c_2d_1e_2f_1g_2h_1$,
			$a_2b_1c_2d_1e_2f_1g_2h_2$, $a_2b_1c_2d_1e_2f_2g_1h_1$, $a_2b_1c_2d_1e_2f_2g_1h_2$,
			$a_2b_1c_2d_1e_2f_2g_2h_1$, $a_2b_1c_2d_1e_2f_2g_2h_2$, $a_2b_1c_2d_2e_1f_1g_1h_1$,
			$a_2b_1c_2d_2e_1f_1g_1h_2$, $a_2b_1c_2d_2e_1f_1g_2h_1$, $a_2b_1c_2d_2e_1f_1g_2h_2$,

n	Symbology	Number	Treatments that will be analyzed
			$a_2b_1c_2d_2e_1f_2g_1h_1$, $a_2b_1c_2d_2e_1f_2g_1h_2$, $a_2b_1c_2d_2e_1f_2g_2h_1$, $a_2b_1c_2d_2e_1f_2g_2h_2$, $a_2b_1c_2d_2e_2f_1g_1h_1$, $a_2b_1c_2d_2e_2f_1g_1h_2$, $a_2b_1c_2d_2e_2f_1g_2h_1$, $a_2b_1c_2d_2e_2f_1g_2h_2$, $a_2b_1c_2d_2e_2f_2g_1h_1$, $a_2b_1c_2d_2e_2f_2g_1h_2$, $a_2b_1c_2d_2e_2f_2g_2h_1$, $a_2b_1c_2d_2e_2f_2g_2h_2$, $a_2b_2c_1d_1e_1f_1g_1h_1$, $a_2b_2c_1d_1e_1f_1g_1h_2$, $a_2b_2c_1d_1e_1f_1g_2h_1$, $a_2b_2c_1d_1e_1f_1g_2h_2$, $a_2b_2c_1d_1e_1f_2g_1h_1$, $a_2b_2c_1d_1e_1f_2g_1h_2$, $a_2b_2c_1d_1e_1f_2g_2h_1$, $a_2b_2c_1d_1e_1f_2g_2h_2$, $a_2b_2c_1d_1e_2f_1g_1h_1$, $a_2b_2c_1d_1e_2f_1g_1h_2$, $a_2b_2c_1d_1e_2f_1g_2h_1$, $a_2b_2c_1d_1e_2f_1g_2h_2$, $a_2b_2c_1d_1e_2f_2g_1h_1$, $a_2b_2c_1d_1e_2f_2g_1h_2$, $a_2b_2c_1d_1e_2f_2g_2h_1$, $a_2b_2c_1d_1e_2f_2g_2h_2$, $a_2b_2c_1d_2e_1f_1g_1h_1$, $a_2b_2c_1d_2e_1f_1g_1h_2$, $a_2b_2c_1d_2e_1f_1g_2h_1$, $a_2b_2c_1d_2e_1f_1g_2h_2$, $a_2b_2c_1d_2e_1f_2g_1h_1$, $a_2b_2c_1d_2e_1f_2g_1h_2$, $a_2b_2c_1d_2e_1f_2g_2h_1$, $a_2b_2c_1d_2e_1f_2g_2h_2$, $a_2b_2c_1d_2e_2f_1g_1h_1$, $a_2b_2c_1d_2e_2f_1g_1h_2$, $a_2b_2c_1d_2e_2f_1g_2h_1$, $a_2b_2c_1d_2e_2f_1g_2h_2$, $a_2b_2c_1d_2e_2f_2g_1h_1$, $a_2b_2c_1d_2e_2f_2g_1h_2$, $a_2b_2c_1d_2e_2f_2g_2h_1$, $a_2b_2c_1d_2e_2f_2g_2h_2$, $a_2b_2c_2d_1e_1f_1g_1h_1$, $a_2b_2c_2d_1e_1f_1g_1h_2$, $a_2b_2c_2d_1e_1f_1g_2h_1$, $a_2b_2c_2d_1e_1f_1g_2h_2$, $a_2b_2c_2d_1e_1f_2g_1h_1$, $a_2b_2c_2d_1e_1f_2g_1h_2$, $a_2b_2c_2d_1e_1f_2g_2h_1$, $a_2b_2c_2d_1e_1f_2g_2h_2$, $a_2b_2c_2d_1e_2f_1g_1h_1$, $a_2b_2c_2d_1e_2f_1g_1h_2$, $a_2b_2c_2d_1e_2f_1g_2h_1$, $a_2b_2c_2d_1e_2f_1g_2h_2$, $a_2b_2c_2d_1e_2f_2g_1h_1$, $a_2b_2c_2d_1e_2f_2g_1h_2$, $a_2b_2c_2d_1e_2f_2g_2h_1$, $a_2b_2c_2d_1e_2f_2g_2h_2$, $a_2b_2c_2d_2e_1f_1g_1h_1$, $a_2b_2c_2d_2e_1f_1g_1h_2$, $a_2b_2c_2d_2e_1f_1g_2h_1$, $a_2b_2c_2d_2e_1f_1g_2h_2$, $a_2b_2c_2d_2e_1f_2g_1h_1$, $a_2b_2c_2d_2e_1f_2g_1h_2$, $a_2b_2c_2d_2e_1f_2g_2h_1$, $a_2b_2c_2d_2e_1f_2g_2h_2$, $a_2b_2c_2d_2e_2f_1g_1h_1$, $a_2b_2c_2d_2e_2f_1g_1h_2$, $a_2b_2c_2d_2e_2f_1g_2h_1$, $a_2b_2c_2d_2e_2f_1g_2h_2$, $a_2b_2c_2d_2e_2f_2g_1h_1$, $a_2b_2c_2d_2e_2f_2g_1h_2$, $a_2b_2c_2d_2e_2f_2g_2h_1$, $a_2b_2c_2d_2e_2f_2g_2h_2$

Statistical models

To construct the statistical models used in data analysis, only the capital letters used to represent the main factors and those corresponding to their possible interactions, a specific value of n should be chosen, then the subscripts that identify them are chosen, and μ, R and ε are added. Their construction was based on the procedures described by Sahagún (1998). For example, in a 2^3 factorial experiment, if Y is the quantitative variable of interest: $Y_{ijkl} = \mu + A_i + B_j + C_k + (AB)_{ij} + (AC)_{ik} + (BC)_{jk} + (ABC)_{ijk} + R_l + \varepsilon_{ijkl}$.

Where: μ is the arithmetic mean of the abcr data; A_i, B_j and C_k are the effects of the i-th, j-th or k-th level of A, B or C, respectively; $(AB)_{ij}$, $(AC)_{ik}$, $(BC)_{jk}$ and $(ABC)_{ijk}$ are the interactions between two or three factors; R_l is the effect of the l-th repetition and ε_{ijkl} is the residual of the model. Table 2 shows the statistical models that will be used to analyze the data when n= 2, 3, 8.

Table 2. Statistical models used in data analysis with InfoStat and InfoGen.

Value of n	Statistical model
2	$Y_{ijk} = \mu + A_i + B_j + (AB)_{ij} + R_k + \varepsilon_{ijk}$
3	$Y_{ijkl} = \mu + A_i + B_j + C_k + (AB)_{ij} + (AC)_{ik} + (BC)_{jk} + (ABC)_{ijk} + R_l + \varepsilon_{ijkl}$
4	$Y_{ijklm} = \mu + A_i + B_j + C_k + D_l + (AB)_{ij} + (AC)_{ik} + (AD)_{il} + (BC)_{jk} + (BD)_{jl} + (CD)_{kl} + (ABC)_{ijk} + (ABD)_{ijl} + (ACD)_{ikl} + (BCD)_{jkl} + (ABCD)_{ijkl} + R_m + \varepsilon_{ijklm}$
5	$Y_{ijklmn} = \mu + A_i + B_j + C_k + D_l + E_m$ + {interactions between two, three, four and five factors}$^{\&}$ + $R_n + \varepsilon_{ijklmn}$
6	$Y_{ijklmno} = \mu + A_i + B_j + C_k + D_l + E_m + F_n$ + {interactions between two, three, four, five and six factors}$^{\&}$ + $R_o + \varepsilon_{ijklmno}$
7	$Y_{ijklmnop} = \mu + A_i + B_j + C_k + D_l + E_m + F_n + G_o$ + {interactions between two, three, four, five, six and seven factors}$^{\&}$ + $R_p + \varepsilon_{ijklmnop}$
8	$Y_{ijklmnopq} = \mu + A_i + B_j + C_k + D_l + E_m + F_n + G_o + H_p$ + {interactions between two, three, four, five, six, seven and eight factors}$^{\&}$ + $R_q + \varepsilon_{ijklmnopq}$
2 to 8	$Y_{ij} = \mu + \tau_i + R_j + \varepsilon_{ij}$ Where: $\tau_i = 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$, or 2^8 treatments

$^{\&}$= the possible interactions for n= 5, 6, 7 and 8 are shown below.

To calculate how many interactions will be indicated in the statistical models, the combinations that are possible for the n factors taken k times are simply added, such that $2 \leq k \leq 8$; k represents, in that order, interactions between two, three, eight factors. If n= 8: $C_2^8 + C_3^8 + C_4^8 + C_5^8 + C_6^8 + C_7^8 + C_8^8 = 28 + 56 + 70 + 56 + 28 + 8 + 1 = 247$ interactions. To verify: $2^n - C_0^n - C_1^n = 2^8 - C_0^8 - C_1^8 = 256 - 1 - 8 = 247$ interactions.

Interactions for n= 8

Between two factors: A*B£, A*C£, A*D£, A*E£, A*F, A*G, A*H, B*C£, B*D£, B*E£, B*F, B*G, B*H, C*D£, C*E£, C*F, C*G, C*H, D*E£, D*F, D*G, D*H, E*F, E*G, E*H, F*G, F*H, G*H.

Between three factors: A*B*C£, A*B*D£, A*B*E£, A*B*F, A*B*G, A*B*H, A*C*D£, A*C*E£, A*C*F, A*C*G, A*C*H, A*D*E£, A*D*F, A*D*G, A*D*H, A*E*F, A*E*G, A*E*H, A*F*G, A*F*H, A*G*H, B*C*D£, B*C*E£, B*C*F, B*C*G, B*C*H, B*D*E£, B*D*F, B*D*G, B*D*H, B*E*F, B*E*G, B*E*H, B*F*G, B*F*H, B*G*H, C*D*E£, C*D*F, C*D*G, C*D*H, C*E*F, C*E*G, C*E*H, C*F*G, C*F*H, C*G*H, D*E*F, D*E*G, D*E*H, D*F*G, D*F*H, D*G*H, E*F*G, E*F*H, E*G*H, F*G*H.

Between four factors: A*B*C*D£, A*B*C*E£, A*B*C*F, A*B*C*G, A*B*C*H, A*B*D*E£, A*B*D*F, A*B*D*G, A*B*D*H, A*B*E*F, A*B*E*G, A*B*E*H, A*B*F*G, A*B*F*H, A*B*G*H, A*C*D*E£, A*C*D*F, A*C*D*G, A*C*D*H, A*C*E*F, A*C*E*G, A*C*E*H, A*C*F*G, A*C*F*H, A*C*G*H, A*D*E*F, A*D*E*G, A*D*E*H, A*D*F*G, A*D*F*H, A*D*G*H, A*E*F*G, A*E*F*H, A*E*G*H, A*F*G*H, B*C*D*E£, B*C*D*F, B*C*D*G,

B*C*D*H, B*C*E*F, B*C*E*G, B*C*E*H, B*C*F*G, B*C*F*H, B*C*G*H, B*D*E*F, B*D*E*G, B*D*E*H, B*D*F*G, B*D*F*H, B*D*G*H, B*E*F*G, B*E*F*H, B*E*G*H, B*F*G*H, C*D*E*F, C*D*E*G, C*D*E*H, C*D*F*G, C*D*F*H, C*D*G*H, C*E*F*G, C*E*F*H, C*E*G*H, C*F*G*H, D*E*F*G, D*E*F*H, D*E*G*H, D*F*G*H, E*F*G*H.

Between five factors: A*B*C*D*E[£], A*B*C*D*F, A*B*C*D*G, A*B*C*D*H, A*B*C*E*F, A*B*C*E*G, A*B*C*E*H, A*B*C*F*G, A*B*C*F*H, A*B*C*G*H, A*B*D*E*F, A*B*D*E*G, A*B*D*E*H, A*B*D*F*G, A*B*D*F*H, A*B*D*G*H, A*B*E*F*G, A*B*E*F*H, A*B*E*G*H, A*B*F*G*H, A*C*D*E*F, A*C*D*E*G, A*C*D*E*H, A*C*D*F*G, A*C*D*F*H, A*C*D*G*H, A*C*E*F*G, A*C*E*F*H, A*C*E*G*H, A*C*F*G*H, A*D*E*F*G, A*D*E*F*H, A*D*E*G*H, A*D*F*G*H, A*E*F*G*H, B*C*D*E*F, B*C*D*E*G, B*C*D*E*H, B*C*D*F*G, B*C*D*F*H, B*C*D*G*H, B*C*E*F*G, B*C*E*F*H, B*C*E*G*H, B*C*F*G*H, B*D*E*F*G, B*D*E*F*H, B*D*E*G*H, B*D*F*G*H, B*E*F*G*H, C*D*E*F*G, C*D*E*F*H, C*D*E*G*H, C*D*F*G*H, C*E*F*G*H, D*E*F*G*H.

Between six factors: A*B*C*D*E*F, A*B*C*D*E*G, A*B*C*D*E*H, A*B*C*D*F*G, A*B*C*D*F*H, A*B*C*D*G*H, A*B*C*E*F*G, A*B*C*E*F*H, A*B*C*E*G*H, A*B*C*F*G*H, A*B*D*E*F*G, A*B*D*E*F*H, A*B*D*E*G*H, A*B*D*F*G*H, A*B*E*F*G*H, A*C*D*E*F*G, A*C*D*E*F*H, A*C*D*E*G*H, A*C*D*F*G*H, A*C*E*F*G*H, A*D*E*F*G*H, B*C*D*E*F*G, B*C*D*E*F*H, B*C*D*E*G*H, B*C*D*F*G*H, B*C*E*F*G*H, B*D*E*F*G*H, C*D*E*F*G*H.

Between seven factors: A*B*C*D*E*F*G, A*B*C*D*E*F*H, A*B*C*D*E*G*H, A*B*C*D*F*G*H, A*B*C*E*F*G*H, A*B*D*E*F*G*H, A*C*D*E*F*G*H, B*C*D*E*F*G*H.

Between eight factors: A*B*C*D*E*F*G*H

The above interactions can also be used to construct any statistical model, such as those corresponding to the series of experiments in randomized complete blocks in arrangement of subdivided plots (Padilla *et al*., 2019; González *et al*., 2019). For example, the 26 interactions that originate between factors A, B, C, D and E, and that must be fed in specifications to the model of InfoStat and InfoGen, were previously identified with the superscript £.

Some formulas for verifying sums of squares with n= 8

The formulas presented below were obtained with the procedures described by Sahagún (1998).

$$\text{SC Factor A} = \frac{\sum_{i=1}^{a} Y_{i}^{2}}{bcdefghr} - \frac{\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq}\right)^{2}}{abcdefghr}$$

$$\text{SC Factor B} = \frac{\sum_{j=1}^{b} Y_{.j.}^{2}}{acdefghr} - \frac{\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq}\right)^{2}}{abcdefghr}$$

$$\text{SC Factor C} = \frac{\sum_{k=1}^{c} Y_{..k}^{2}}{abdefghr} - \frac{\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq}\right)^{2}}{abcdefghr}$$

$$SC\ Factor\ D = \frac{\sum_{l=1}^{d} Y^2_{...l....}}{abcefghr} - \frac{(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq})^2}{abcdefghr}$$

$$SC\ Factor\ E = \frac{\sum_{m=1}^{e} Y^2_{....m....}}{abcdfghr} - \frac{(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq})^2}{abcdefghr}$$

$$SC\ Factor\ F = \frac{\sum_{n=1}^{f} Y^2_{.....n...}}{abcdeghr} - \frac{(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq})^2}{abcdefghr}$$

$$SC\ Factor\ G = \frac{\sum_{o=1}^{g} Y^2_{......o..}}{abcdefhr} - \frac{(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq})^2}{abcdefghr}$$

$$SC\ Factor\ H = \frac{\sum_{p=1}^{h} Y^2_{.......p.}}{abcdefgr} - \frac{(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq})^2}{abcdefghr}$$

$$SC\ Repetitions = \frac{\sum_{q=1}^{r} Y^2_{........q}}{abcdefgh} - \frac{(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h}\sum_{q=1}^{r} Y_{ijklmnopq})^2}{abcdefghr}$$

Next, the second part of each formula, which corresponds to the correction factor, will be identified as FC and for convenience, the SCs of treatments 1, 2, s will be defined so that by difference the SC of the interaction of interest is calculated. The procedure is similar to that reported in Padilla *et al*. (2019), for a series of experiments in randomized complete blocks in arrangement of subdivided plots.

SC for an interaction between two factors

SC Treatments 1 = SC A + SC B + SC A∗B

SC A∗B = SC Treatments 1 – SC A – SC B

Where: $SC\ Treatments\ 1 = \frac{\sum_{i=1}^{a}\sum_{j=1}^{b} Y^2_{ij.......}}{cdefghr} - FC.$

SC for an interaction between three factors

SC A∗B∗C = SC Treatments 2 - SC A - SC B - SC C - SC A∗B - SC A∗C - SC B∗C

Where: $SC\ Treatments\ 2 = \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} Y^2_{ijk......}}{defghr} - FC.$

SC for an interaction between four factors

SC A∗B∗C∗D = SC Treatments 3 - SC A - SC B - SC C - SC D - SC A∗B - SC A∗C - SC A∗D - SC B∗C - SC B∗D - SC C∗D - SC A∗B∗C - SC A∗B∗D - SC A∗C∗D. Where: SC Treatments 3 = $\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d} Y^2_{ijkl.....}}{efghr} - FC.$

SC for an interaction between five factors

SC A∗B∗C∗D∗E = SC Treatments 4 - SC A - SC B - SC C - SC D - SC E - (SC interactions between two, three and four factors). Where: SC Treatments 4 = $\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e} Y^2_{ijklm....}}{fghr} - FC.$ Henceforth, the SCs for A, B, C, D, E, F, G, H will be identified as SC main factors.

SC for an interaction between six factors

SC A∗B∗C∗D∗E∗F = SC Treatments 5 - SC main factors - SC interactions between two, three, four and five factors. Where: SC Treatments 5 = $\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f} Y_{ijklmn\ldots}^{2}}{ghr}$ - FC.

SC for an interaction between seven factors

SC A∗B∗C∗D∗E∗F∗G = SC Treatments 6 - SC main factors - SC interactions between two, three, four, five and six factors. Where: SC Treatments 6 = $\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g} Y_{ijklmno..}^{2}}{hr}$ - FC.

SC for the interaction between eight factors

SC A∗B∗C∗D∗E∗F∗G∗H = SC Treatments 7 - SC main factors - SC interactions between two, three, four, five, six and seven factors. Where: SC Treatments 7 = $\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{m=1}^{e}\sum_{n=1}^{f}\sum_{o=1}^{g}\sum_{p=1}^{h} Y_{ijklmnop.}^{2}}{r}$ - FC.

Development of databases

These must have the structure of Table 3, in case the partial or total number of data captured is observed, Rep identifies repetitions, Trat defines the treatment, Ren specifies the yield and in A, B, H, the combinations between both levels of each of the eight factors are recorded.

Table 3. Structure of the database of a 2^8 factorial experiment.

Case	Rep	Trat	A	B	C	D	E	F	G	H	Ren
1	1	01	1	1	1	1	1	1	1	1	4
2	1	02	1	1	1	1	1	1	1	2	6.3
3	1	03	1	1	1	1	1	1	2	1	5.3
4	1	04	1	1	1	1	1	1	2	2	6.9
5	1	05	1	1	1	1	1	2	1	1	6.5
6	1	06	1	1	1	1	1	2	1	2	7.9
7	1	07	1	1	1	1	1	2	2	1	7.2
8	1	08	1	1	1	1	1	2	2	2	8.3
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
m-1	r	t-1	a-1	b-1	c-1	d-1	e-1	f-1	g-1	h-1	8.6
m	r	t	a	b	c	d	e	f	g	h	9.1

m= 2^nr observations, r= number of repetitions; Rep= repetition; Trat= treatment; Ren= yield (t ha^{-1}).

Saving this treatment structure follows the same recommendations as for any file that contains databases, the file could be called EXP2N. IDB2.

Procedure for analyzing the data

After loading InfoStat or InfoGen, two dialog boxes are displayed on the screen, in both choose ok, their main menu will appear. In file choose the open option to load the database. InfoStat or InfoGen will display a table similar to that of Table 3.

In statistics choose analysis of variance and another dialog box will appear, where the definition of the dependent and classification variables is requested. If the objective is to analyze the 2^8 factorial experiment, the dependent variable is rinde, the classification variables are Rep, A, B, C, D, E, F, G, H. Click accept.

Another dialog box that says analysis of variance will be displayed on the screen. Here, the user will verify that in the specifications of the terms of the model, the classification variables (Rep, A, B, H) are correct, the interactions A∗B, A∗C, A∗B∗C∗D∗E∗F∗G∗H must be entered below the last main factor. In a 2^3 factorial experiment, Rep, A, B, C, A∗B, A∗C, B∗C, A∗B∗C should be displayed vertically in the dialog box. It is not necessary to specify either the arithmetic mean or the residual of the model. The analysis of variance will be obtained.

In the last dialog box, a comparison of treatment means with LSD Fisher, Bonferroni, Tukey, Duncan and Scheffé, among others, could be chosen. In comparisons choose: Tukey/show means according to: Rep, A, B, C, H, A∗B, A∗C, A∗B∗C∗D∗E∗F∗G∗H/ presentation in descending list/significance level 0.01/accept. This procedure generates the analysis of variance and the comparison of means with the Tukey test (p= 0.01) for the eight factors and for their 247 interactions.

Conclusions

If the user wants to perform an analysis of variance using the general statistical model and a comparison of means, in the corresponding dialog box, in classification variables they must choose rep trat and rinde. InfoStat and InfoGen will only show the output with the 256 treatments for a randomized complete block design, if in classification variables rep is not chosen, then the analysis of variance of a completely randomized design will be obtained.

Cited literature

Álvarez, G. L.; Guerra, S. J. y Rojas, R. N. 2018. Diseño factorial 2^k aplicado a la caracterización reológica de suspensiones de caolín. Prospectiva. 16(1):18-25.

Balzarini, M. G.; González, L.; Tablada, M.; Casanoves, F.; Di Rienzo, J. A. y Robledo, C. W. 2008. InfoStat, versión 2008. Manual del usuario. Editorial Brujas, Córdoba, Argentina. 336 p.

Balzarini, M.; Bruno, C. y Arroyo, A. 2016. Análisis de ensayos agrícolas multi-ambientales. ejemplos con Info-Gen. Manual del Usuario. Unidad de Estadística y Biometría. Facultad de Ciencias Agropecuarias. Universidad Nacional de Córdoba, Argentina. 141 p.

Balzarini, M. G. y Di Rienzo, J. A. 2016. InfoGen, versión 2016. FCA. Universidad Nacional de Córdoba, Argentina. www.info-gen.com.ar.

Di Rienzo, J. A.; Casanoves, F.; Balzarini, M. G.; González, L.; Tablada, M. y Robledo, C. W. 2008. InfoStat, versión 2008. Grupo InfoStat, FCA. Universidad Nacional de Córdoba, Argentina. 336 p.

Gomez, K. A. and Gomez, A., A. 1984. Statistical procedures for agricultural research. 2nd (Ed). John Wiley and Sons, Inc. Printed in Singapore. 680 p.

González, H. A.; Pérez, L. D. J.; Rubí, A. M.; Gutiérrez, R., F.; Franco, M. J. R.P. y Padilla, L. A. 2019. InfoStat, InfoGen y SAS para contrastes mutuamente ortogonales en experimentos en bloques completos al azar en parcelas subdivididas. Rev. Mex. Cienc. Agríc. 10(6):1417-1431.

Jiménez, C. M. G. 2015. Superficies de respuesta mediante un diseño central compuesto. Instituto de Estadística Teórica y Aplicada. Rev. Varianza. 11(1):31-36.

Martínez, G. A. 1988. Diseño de experimentos. Teoría y métodos. Editorial Trillas, 1ra (Ed). México, DF. 756 p.

Medina, V. P. D. y López, R. A. M. 2011. Análisis crítico del diseño factorial 2^k sobre casos aplicados. Scientia Et Technica XVII. 47(1):101-106.

Montgomery, D. C. 2010. Diseño y análisis de experimentos. Editorial Limusa SA de CV. Grupo Noriega Editores. 2nd (Ed). México, DF. 686 p.

Padilla, L. A.; González, H. A.; Pérez, L. D. J.; Rubí, A. M.; Gutiérrez, R. F.; Ramírez, D. J. F.; Franco, M. J. R. P. y Serrato, C. R. 2019. Programas para SAS e InfoStat para analizar una serie de experimentos en parcelas subdivididas. Temas Selectos en la Innovación de las Ciencias Agropecuarias. 1. Agricultura. 2. Desarrollo Rural. Alfaomega Grupo Editor S.A. de C.V. 1ra (Ed). México, DF. 67-88 pp. (ISBN:978-607-538-411-5).

Quiroz, M. J.; Pérez, L. D. J.; González, H. A.; Rubí, A. M.; Gutiérrez, R. F.; Franco, M. J. R. P. y Ramírez, D. J. F. 2017. Respuesta de 10 cultivares de maíz a la densidad de población en tres localidades del centro mexiquense. Rev. Mex. Cienc. Agríc. 8(7):1521-1535.

Reynoso, Q. C. A.; González, H. A.; Pérez, L. D. J.; Franco, M. O.; Torres, F. J. L.; Velázquez, C. G. A.; Breton, L. C.; Balbuena, M. A. y Mercado, V. O. 2014. Análisis de 17 híbridos de maíz sembrados en 17 ambientes de los Valles Altos del centro de México. Rev. Mex. Cienc. Agríc. 5(5):871-882.

Rodríguez, F. I.; González, H. A.; Pérez, L. D. J. y Rubí, A. M. 2015. Efecto de cinco densidades de población en ocho cultivares de maíz sembrados en tres localidades del Valle de Toluca, México. Rev. Mex. Cienc. Agríc. 6(8):1943-1955.

Sahagún, C. J. 1998. Construcción y análisis de los modelos fijos, aleatorios y mixtos. Universidad Autónoma Chapingo (UACH)-Departamento de Fitotecnia. Boletín técnico núm. 2. 65 p.

SAS. 1989. Institute Inc. SAS/STAT User’s Guide, version 6. Fourth edition, Volume 1, Cary, NC. USA. 943 p.

Torres, F. J. L.; Morales, R. E. J.; González, H. A.; Laguna, C. A. y Córdova, O. H. 2011. Respuesta de híbridos trilineales y probadores de maíz en Valles Altos del centro de México. Rev. Mex. Cienc. Agríc. 2(6):829-844.

Torres, F. J. L.; Mendoza, G. B.; Prassana, B. M.; Alvarado, G.; San Vicente, E. M. and Crossa, J. 2017. Grain yield and stability of white early maize hibrids in the higland valleys of Mexico. Crop Sci. 57(6):3002-3015.