Article

Deterministic equation for hydraulic system design multiple outlet irrigation

José Guadalupe González-Quirino[1]
Manuel Fortis-Hernández[1]
Pablo Preciado-Rangel[1]
Pablo Yescas-Coronado[1]
Juan Manuel Barrios-Díaz[3]
Arturo Reyes-González[2§]

[1]National Technology of Mexico-*Campus* Technological Institute of Torreón (TecNm-ITT). Old road Torreón-San Pedro km 7.5, Torreón, Coahuila, Mexico. ZC. 27170. Tel. 871 7507199. (gonzalezquirino@hotmail.co; pyescas@hotmail.com; textrc2010@live.com; ppreciador@yahoo.com.mx).
[2]INIFAP. Boulevard José Santos Valdez 1200, Matamoros, Coahuila. (reyes.arturo@inifap.gob.mx).
[3]Meritorious Autonomous University of Puebla-Faculty of Agrohydraulic Engineering. University Avenue s/n, San Juan Acateno, Teziutlan, Puebla. ZC. 73965. Tel. 231 3122933. (juan.barrios@correo.buap.mx).

[§]Corresponding author: reyes.arturo@inifap.gob.mx.

Abstract

In the design of multi-outlet irrigation systems, it is necessary to calculate the frictional energy losses in pipes and determine the most economical and optimal diameter, for this, there are the equations of Manning, Hanzen & Williams and Darcy & Wesbach. According to the above, the objective of this work was to perform a mathematical physical analysis of the equations of energy losses due to friction in pipes and fittings to generate at least one deterministic equation based on the Manning and Darcy & Wesbach equation for the hydraulic design of multi-outlet irrigation and reduce high costs in the operation of agricultural irrigation equipment. The study was conducted in spring 2021, at the National Technological Institute of Mexico, Technological Institute of Torreón campus. The equations of localized and frictional energy losses (Darcy & Wesbach and Manning) were algebraically merged, and three deterministic equations were obtained for the design of multiple outlets with a relative error range from -0.009 to 0.209% in relation to the equation proposed by SARH 1979 (CENAMAR).

Keywords: Darcy & Wesbach, deterministic equation, energy losses, Manning.

Reception date: April 2021
Acceptance date: July 2021

Introduction

In the design of multi-outlet irrigation systems, there is a need to calculate energy losses due to friction in pipelines (Pimenta *et al*., 2018) and determine the most economical and optimal diameter (Espinosa *et al*., 2016; Melo *et al*., 2019). This loss is due to friction between the particles of the fluid and the wall of the pipeline (Nosrati *et al*., 2017), for this, there are the equations of Manning, Hanzen & Williams and Darcy & Wesbach. These equations have a coefficient of multiple outlets or a factor of outlets or friction that makes it difficult to calculate the number of drip emitters (emitters).

In addition, the Hanzen & Williams equation overestimates the costs of frictional energy losses by 13% compared to Darcy & Wesbach (Flecha *et al*., 2010), which makes the operation of irrigation equipment more expensive. Alegret and Martínez (2019) proposed a coefficient to the Hazen & Williams equation based on relative roughness and the Reynolds number, obtaining values very similar to the energy losses of the Darcy & Wesbach equation. Adams (2016) combined the roughness of grains of sand with the Darcy &Wesbach equation and the Coolebrock & White equation and determined the roughness coefficient of Hazen & Williams with a deviation of 2.3%.

Jamil and Mujeebu (2019); Jamil (2019) determined an equation to calculate energy losses with Darcy & Wesbach without the use of the friction factor and Reynolds number. Jiménez and Ramírez (2018) combined the Manning and Darcy &Wesbach equation with the Christiansen multiple outlets factor to determine the number of outlets using the Newton Raphson numerical method. Wang *et al*. (2016) performed a hydraulic analysis of a drip irrigation unit with finite elements and an iterative Gaussian elimination method, and it determined that a multi-outlet irrigation line is an alternative to a branched system. Hassan (2017) developed a mathematical model of linear programming in which he evaluated the costs and losses of energy in different pipe diameters and determined that the best equations for energy losses were those of Darcy & Wesbach and Manning. A mathematical physical analysis of the equations of frictional energy losses in pipes and fittings was performed to generate at least one deterministic equation based on the Manning and Darcy &Wesbach equation for the hydraulic design of multi-outlet irrigation.

Materials and methods

This work was carried out in the facilities of the Technological Institute of Torreón, located in the Ejido Ana, Torreón San Pedro Road kilometer 7.5, in the parallels 25° 36’ 53’’ of north latitude and 103° 22’ 21’’ of west longitude.

Energy loss equation

Hassan (2017) developed a mathematical model of linear programming in which he evaluated the costs and losses of energy in five nominal diameters of pipelines 3”, 3-1/2”, 4”, 5” and 6” with the equations of Hazen-Williams, Manning, Scobey and Darcy & Wesbach and determined that the best equations for energy losses were those of Darcy & Wesbach and Manning. For this reason, in this work the Manning and Darcy & Wesbach equations were used.

Frictional energy loss equation

Next, Manning's frictional energy loss equation 1 is presented $Hf=\frac{4^{\frac{10}{3}}n^2Lq^2}{\pi^2 d^{\frac{16}{3}}}$ 1). Where: Hf= energy loss due to friction (m); q= pipe flow (m^3 s^{-1}); L= pipe length (m); n= Manning's roughness coefficient; and d= pipe diameter (m).

Localized energy loss equation

The Darcy & Wesbach localized energy loss equation. $HL=K_L\frac{v^2}{2g}$ 2). Where: HL= localized energy loss (m); K_L= localized energy loss coefficient; v= velocity of the fluid (m); and g= acceleration of gravity (m s^{-2}). If the velocity is substituted in the previous equation as a function of flow and area, it results in: $HL=K_L\frac{8(q)^2}{g\pi^2 d^4}$ 3).

Analysis of an irrigating line

Figure 1 shows an irrigating line of emitters. When analyzing the irrigating line from right to left, a flow 'q' circulates in the section of pipe one, a flow '2q' circulates in section 2, a flow '3q' circulates in section 3, a flow '4q' circulates in section 4 and so on. This means that the flow increases by a 'q' in each section of pipe until the maximum expenditure of the pipe 'Nq' in the 'NL' section is reached. Therefore, frictional and localized energy losses are accumulating in a 'q' in each section of pipe through which the water circulates until having the total energy losses in the 'NL' section and maximum flow 'Nq'.

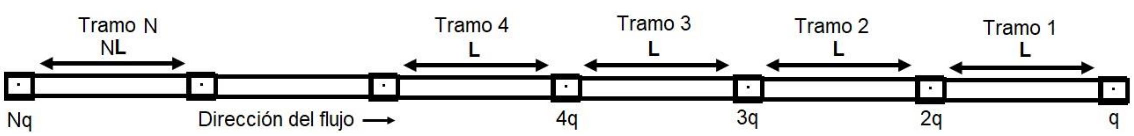

Figure 1. Diagram of an irrigating line of drippers.

By adding the energy losses due to friction in each section from right to left, using equation 1, gives equation (4). $Hf_T=\frac{4^{\frac{10}{3}}n^2L(1q)^2}{\pi^2 d^{\frac{16}{3}}}+\frac{4^{\frac{10}{3}}n^2L(2q)^2}{\pi^2 d^{\frac{16}{3}}}+\frac{4^{\frac{10}{3}}n^2L(3q)^2}{\pi^2 d^{\frac{16}{3}}}.......+\frac{4^{\frac{10}{3}}n^2L(Nq)^2}{\pi^2 d^{\frac{16}{3}}}=\frac{4^{\frac{10}{3}}n^2L(q)^2}{\pi^2 d^{\frac{16}{3}}}\sum_{i=1}^{N} i^2$ 4). By adding the localized energy losses in each emitter from right to left, with equation (3), equation (5) results. $HL_T=K_L\frac{8(1q)^2}{g\pi^2 d^4}+K_L\frac{8(2q)^2}{g\pi^2 d^4}+K_L\frac{8(3q)^2}{g\pi^2 d^4}+K_L\frac{8(4q)^2}{g\pi^2 d^4}+............K_L\frac{8(Nq)^2}{g\pi^2 d^4}=K_L\frac{8(q)^2}{g\pi^2 d^4}\sum_{i=1}^{N} i^2$ 5). By adding the total energy losses due to friction 'Hf_T' (4), total localized energy losses HL_T (5) and if factored, the equation of total energy losses in an irrigating line (6) results. $f_{TLR}=\left(\frac{4^{\frac{10}{3}}n^2Lq^2}{\pi^2 d^{\frac{16}{3}}}+K_L\frac{8q^2}{g\pi^2 d^4}\right)\sum_{i=1}^{N} i^2$ 6).

Solving the sum of equation (6) and substituting it into the same equation, equation (7) results. $\left(\frac{4^{\frac{10}{3}} n^2 L q^2}{\pi^2 d^{\frac{16}{3}}} + K_L \frac{8q^2}{g\pi^2 d^4}\right)\left(\frac{2N^3+3N^2+N}{6}\right) = Hf_{Total}$ 7). Arranging terms “N” from the above equation, a polynomial of the total energy of the irrigating line results the equation (8). $2N^3+3N^2+N-\frac{6Hf_{Total}}{\left(\frac{4^{\frac{10}{3}} n^2 L q^2}{\pi^2 d^{\frac{16}{3}}} + K_L \frac{8q^2}{g\pi^2 d^4}\right)} = 0$ 8).

Solution of the polynomial of the irrigating line energy

The constant term is extracted from equation eight and if analyzed algebraically, equation 8.1 results. $K= \frac{3/4 * Hf_{Total} * \pi^2 g d^{\frac{16}{3}}}{4^{\frac{11}{6}} g n^2 s q^2 + K_L q^2 d^{\frac{4}{3}}}$ 8.1). Compacting equation 8, it transforms into equation 9. $F(N)=2N^3+3N^2+N-K= 0$ 9). The above equation consists of the following coefficients: a=2, b=3, c=1 and d= -K.

Next, equation (10) is presented, which solves equation (9) by generating the real solution of the equation. $N= \left[\frac{K}{4} + \sqrt{\frac{3ac-b^2}{9a^2} + \left(\frac{K}{4}\right)^2}\right]^{\frac{1}{3}} + \left[\frac{K}{4} - \sqrt{\frac{3ac-b^2}{9a^2} + \left(\frac{K}{4}\right)^2}\right]^{\frac{1}{3}} - \frac{1}{2}$ 10). Substituting the coefficients a, b and c of equation (9) into equation (10), equation (11) is obtained, which is the solution of equation (9). $N= \left[\frac{K}{4} + \sqrt{\left(\frac{K}{4}\right)^2 - \frac{1}{12} +}\right]^{\frac{1}{3}} + \left[\frac{K}{4} - \sqrt{\left(\frac{K}{4}\right)^2 - \frac{1}{12}}\right]^{\frac{1}{3}} - \frac{1}{2}$ 11).

Analysis of the real solution of the polynomial of total energy of the irrigating line

Equation 11 is composed of three terms (1°, 2° and 3°), the first two terms contain a variable and the third term is a constant, these terms are presented below (equations 11.1, 11.2 and 11.3, respectively). $1st= \left[\frac{K}{4} + \sqrt{\left(\frac{K}{4}\right)^2 - \frac{1}{12}}\right]^{\frac{1}{3}}$ 11.1); $2nd= \left[\frac{K}{4} - \sqrt{\left(\frac{K}{4}\right)^2 - \frac{1}{12}}\right]^{\frac{1}{3}}$ 11.2); $3rd= -\frac{1}{2}$ 11.3).

Statistical analysis of the number of outlets

The relative percentage error was determined, for this, the equation of Olivares *et al*. (2019) was adapted, obtaining the following equation. Relative error (%)= $\left(\frac{Ne-N}{N}\right)100$ 12). Where: relative error (%) = is the percent of relative error, N= number of emitters with equation (11); Ne= number of drippers estimated with equation 11.1.

To estimate this error, an electronic spreadsheet (Excel) was used for each of the equations that determine the number of outlets. Table 1 shows the numerical value of ‘N’ number of emitters (11), the error in the percentage of ‘N’ number of emitters in relation to the first term ‘1st’ (11.1), the partial solution of the terms (1°, 2° and 3°) to the total solution of the polynomial, as well as the contribution to the solution in percentage of each of the terms.

This table shows that regarding the contribution of the first term ‘1°’ in the column of partial solutions, the decimal part of these values tends to be constant as N increases and in the column of contribution to the solution, it tends to decrease. The contribution of the second term ‘2°’ in the partial solutions column tends to zero when ‘N’ takes values equal to or greater than 87 and in the column of contribution to the solution, it tends to a small value when ‘N’ is equal to 87 outlets. Likewise, the contribution of the third term ‘3°’ serves as a correction factor to the solution and tends to decrease significantly in the same number of outlets in the column of contribution to the solution. This means that the compact solution of equation (9) is summarized in the first ‘1°’ and third ‘3°’ term of equation (11), resulting in equation 13. $N=\left[\frac{K}{4}+\sqrt{\left(\frac{K}{4}\right)^2-\frac{1}{12}}\right]^{\frac{1}{3}}-\frac{1}{2}$ 13).

Table 1. Analysis of the real solution of the polynomial of total energy of the irrigating line.

K	N	‘N’ error (%)	Partial solutions			Contribution to the solution (%)			
			1°	2°	3°	1°	2°	1°	2°
2.8	1	10.38	1.1	0.4	-0.5	110.38	39.71	-50.09	60.29
25	2	15.52	2.32	0.19	-0.5	115.52	9.37	-24.89	90.63
77	3	12.33	3.38	0.13	-0.5	112.33	4.30	-16.63	95.7
171	4	10.01	4.41	0.1	-0.5	110.01	2.48	-12.49	97.52
319	5	8.38	5.42	0.08	-0.5	108.38	1.61	-9.99	98.39
533	6	7.2	6.44	0.07	-0.5	107.2	1.13	-8.33	98.87
825	7	6.3	7.44	0.06	-0.5	106.3	0.84	-7.14	99.16
1 206	8	5.6	8.45	0.05	-0.5	105.6	0.65	-6.25	99.35
2 288	10	4.58	10.46	0.04	-0.5	104.58	0.42	-5	99.58
10 673	17	2.79	17.48	0.02	-0.5	102.79	0.15	-2.94	99.85
51 268	29	1.67	29.49	0.01	-0.5	101.67	0.05	-1.72	99.95
257 445	50	0.98	50.49	0.01	-0.5	100.98	0.02	-1	99.98
1 339615	87	0.57	87.5	0	-0.5	100.57	0.01	-0.57	99.99
458 199	133	0.37	133.5	0	-0.5	100.37	0	-0.38	100
20 015 200	215	0.23	215.50	0	-0.5	100.23	0	-0.23	100
78 955 400	340	0.15	340.5	0	-0.5	100.15	0	-0.15	100
576 300 000	660	0.08	660.5	0	-0.5	100.08	0	-0.08	100
2 003 002 000	1 000	0.05	1 000.5	0	-0.5	100.05	0	-0.05	100

Data calculated using equations 11.1,11.2, 11.3, 12 and 13. Where: K= constant of the polynomial; N= number of emitters; 1°= first term; 2°= second term; and 3°= third term of the equation.

Equation 13 consists of two variable subterms, which are presented below. Subterm 1= $\frac{K}{4}$ 13.1); Subterm 2= $\sqrt{\left(\frac{K}{4}\right)^2-\frac{1}{12}}$ 13.2).

Table 2 shows the numerical value of 'N' number of emitters (11), the values of 'subterm 1' (13.1) and 'subterm 2' (13.2), the numerical value of 'modified N', calculated with the sum of equations 13.1 and 13.2 that represents equation (11.1); the 'N' error determined with the values of 'modified N' and the values of 'N', calculated with equation (12).

This table shows the contribution of subterm 1 and subterm 2, where it is observed that there is little difference between the two columns when 'N' takes values from one to seven and the values of the two subterms are equal when N is equal to or greater than eight. This is because the subtraction of one twelfth to the second term of the equation does not affect the value of subterm two, comparing the contribution of the two subterms when 'N' takes a value of 87 the 'N' error calculated with equation 12 is 0.6 percent. Therefore, the value of -1/12 of the 'subterm 2' is removed, giving rise to equation (13.21). Subterm 2= $\frac{K}{4}$ 13.21)

Table 2. Analysis of the subterms of equation 21 (real solution of the polynomial of total energy of the irrigating line).

K	N	Subterm 1	Subterm 2	Modified N	'N' error (%)
2.9	1	0.7	0.7	1.1	10.8
25	2	6.3	6.2	2.3	15.5
77	3	19.3	19.2	3.4	12.3
171	4	42.8	42.7	4.4	10
319	5	79.8	79.7	5.4	8.4
533	6	133.3	133.2	6.4	7.2
825	7	206.3	206.2	7.4	6.3
1 206	8	301.5	301.5	8.4	5.6
2 288	10	572	572	10.5	4.6
4 886	13	1 221.5	1 221.5	13.5	3.6
10 673	17	2 668.3	2 668.2	17.5	2.8
51 268	29	12 817	12 817	29.5	1.7
257 445	50	64 361.3	64 361.2	50.5	1
1 339 615	87	334 903.8	334 903.7	87.5	0.6
4 758 199	133	1 189 549.8	1 189 549.7	133.5	0.4
11 761 000	180	2 940 250	2 940 250	180.5	0.3
20 015 200	215	5 003 800	5 003 800	215.5	0.2
78 955 400	340	19 738 850	19 738 850	340.5	0.1
576 300 000	660	144 075 000	144 075 000	660.5	0.1
2 003 002 000	1 000	500 750 500	500 750 500	1 000.5	0

Data calculated using equations 11, 13.1, 13.2 and 12.

By merging subterm 1 (13.1) with subterm 2 (13.21), equation (14) results as follows: N= $\left[\frac{K}{2}\right]^{\frac{1}{3}}$ 14).

Deterministic equations

Joining equation (8.1) with equation (14) results in equation (15). $N=\left[\frac{3/_8(Hf_{Total})\pi^2 gd^{\frac{16}{3}}}{4^{\frac{11}{3}}gn^2sq^2+K_Lq^2d^{\frac{4}{3}}}\right]^{\frac{1}{3}}$ 15). Adding the constant term (11.3), which has a value of (-1/2), to equation (15) results in equation (16). $N=\left[\frac{3/_8(Hf_{Total})\pi^2 gd^{\frac{16}{3}}}{4^{\frac{11}{3}}gn^2sq^2+K_Lq^2d^{\frac{4}{3}}}\right]^{\frac{1}{3}}-\frac{1}{2}$ 16).

Table 3 shows the 'N values' calculated with equations (15) and (16) and their respective relative errors based on equation (11), calculated with equation (12). Equation (16), as N is greater than 87 emitters, errors are zero compared to equation (11), while in equation (15), errors tend to zero from 87 emitters. Equation (16) has a better fit than equation (15), with errors ranging from -37.28% for one emitter and -0.01% for 87 emitters. Equation (15) has a lower fit compared to equation (11), with errors ranging from 12.35% for one emitter and 0.56% for 87 emitters.

Table 3. Relative errors of equations 15 and 16, in relation to equation 11.

K	'N' values in the equations			'N' errors (%)	
	11	15	16	15	16
2.9	1	1.1	0.6	12.35	-37.28
25	2	2.3	1.8	15.54	-9.35
171	4	4.4	3.9	10.01	-2.48
533	6	6.4	5.9	7.2	-1.13
1 206	8	8.4	7.9	5.6	-0.65
2 288	10	10.5	10	4.58	-0.42
10 673	17	17.5	17	2.79	-0.15
51 268	29	29.5	29	1.67	-0.05
257 445	50	50.5	50	0.98	-0.02
1 339 615	87	87.5	88.3	0.56	-0.01
11 761 000	180	180.5	180	0.28	0
78 955 400	340	340.5	340	0.15	0
576 300 000	660	660.5	660	0.08	0
2 003 002 000	1 000	1 000 5	1 000	0.05	0

Data calculated using equations 11, 12, 15 and 16.

Validation of deterministic equations

For the validation of the deterministic equations (11, 15 and 16) it was necessary to design an irrigating line in an Excel spreadsheet. For this purpose, the following data and calculations are presented.

Design of an irrigating line

An irrigating line was designed with the methodology proposed by the National Center of Advanced Irrigation Methods (CANAMAR, 1979) belonging to the Secretariat of Agriculture and Hydraulic Resources (SARH).

Selection of the dripper

A non-self-compensated dripper with a consumption greater than one liter per hour was selected, the selected dripper was that of low spending Rivulis E1000 (Rivulis Irrigation Inc; San Diego, CA, USA), the technical data are presented in Table 4.

Table 4. Hydraulic behavior of the Rivulis E1000 low-spending dripper.

Hydraulic load (m)	Flow (L h^{-1})
8.156	1.84
10.195	2.05
12.234	2.25
14.273	2.42
16.312	2.594
18.351	2.74
20.39	2.864

Rivulis E1000 technical data.

Hydraulic model of the dripper

The technical data of the dripper underwent a potential regression analysis in an Excel spreadsheet, the results are shown in Figure 2.

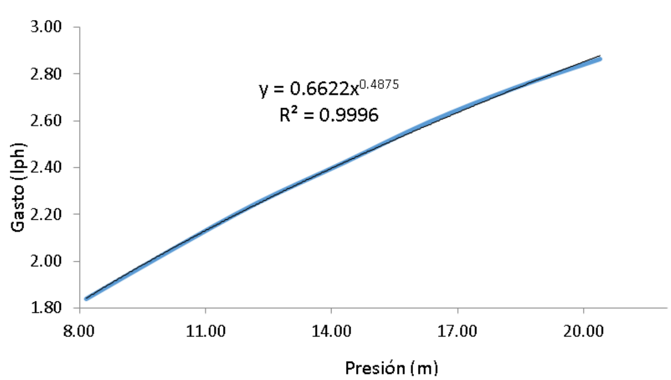

Figure 2. Regression analysis of the dripper (Rivulis E1000) to estimate the values of k and x.

The results allowed determining the components of the hydraulic model of drippers, where the value of coefficient ‘k’ was equal to 0.6622 and the value of ‘x’ was 0.4875 exponent (Figure 2).

Calculation of the operating pressure of the irrigating line

The operating pressure of the dripper irrigation line was also determined with equation (17). $H_0=\left[\frac{qm}{k\left(1-\frac{\%Vg}{100}\right)^{0.5}}\right]^{\frac{1}{x}}$ 17). Where: Ho= hydraulic load of operation of the dripper (m); qm= average flow of the dripper (2 L h^{-1}); %Vg= percent of flow variation between the first and last dripper (5%).

Calculation of permissible energy loss in the irrigating line

The calculation of the permissible energy loss was determined with equation (18). $Dh=H_o\left[1-\left(\frac{100-\%Vg}{100}\right)^{\frac{1}{x}}\right]$ 18). Where: Dh= permissible energy loss (m).

Selection of diameters for the calculation of the number of drippers in the irrigating line

Eight diameters of PVC (polyvinyl chloride) pipeline were selected, for this purpose, the English series Amanco (Orbia Inc; San Francisco CA, USA) was selected because this type of pipeline has semi-equidistant internal diameters (Table 5).

Table 5. Internal diameters of English series Amanco PVC pipeline.

Nominal diameter (mm)	Internal pipe diameters (mm)			
	RD-13.5	RD-26	RD-32.5	RD-41
13	17.5			
25	27.8	29.8		
38	40.5	43.9		
50	50.7	55.1		
75		81.5		
100		104.9	106.7	108.1
150		154.5	157.3	159.5
200		201.3	204.9	207.9

Amanco (San Francisco, USA).

Calculation of the number of drippers in different internal pipe diameters

An irrigating line was designed by determining the number of drippers that can be installed in different internal pipe diameters (Table 5) with equation 19. $N=\left[\frac{Hf_{Total}}{\left(\frac{10.29n^2Lq^2}{d^{\frac{16}{3}}}+\frac{K_Lq^2}{2gA}\right)\left(\frac{1}{3}+\frac{1}{2N}+\frac{1}{6N^2}\right)}\right]^{\frac{1}{3}}$ 19).

Where: N= number of drippers; A= cross-sectional area of the pipe (m^2); K_L= dripper insertion coefficient (0.5); n= roughness coefficient of PVC pipe (0.0079); Hf_{Total}= total energy loss, for this case, a single irrigating line was designed Hf_{Total}= Dh. The number of drippers was calculated with equation 19 and equations 11, 15 and 16. The percent relative error was determined with equation 12 based on equation 19 (Table 6) (statistical analysis of the number of outlets).

Results and discussion

When generating the deterministic equations for the design of multiple outlets (equation 11, 15 and 16), a comprehensive analysis of energy losses in pipes associated with roughness, fittings according to Haddad (2019); Tas *et al*. (2020), and diameters according to Vilaça *et al*. (2017), in the hydraulic design and analysis, the outlet flow of the emitter was taken into account according to Sadeghi *et al*. (2016); likewise, the energy losses in the fittings of the irrigating line were accurately determined as they can vary from 6.3 to 49% (Wang and Chen, 2018). However, Wang *et al*. (2019) report that the relationship of minor and total load loss in a pipeline can reach 71.71%.

According to the literature discrepancies of energy losses in fittings or localized, the equation of localized energy loss (4) was algebraically considered in the generation of the three deterministic equations (11, 15 and 16). Regarding energy losses due to friction in pipes, Monserrat *et al*. (2018) points out that the maximum length of an irrigating line on leveled soil tends to be the same for any slope and facilitates economic optimization, while Wang *et al*. (2018) reported that minor losses and friction losses are closely related to dripper spacing, which is why the frictional energy loss equation (5) was used algebraically.

Table 6 shows the relative errors in percent of the three deterministic equations (11, 15 and 16) compared to equation (19). Equation (11) has a minimum value of -0.009% and a maximum value of 0.006%, equation 15 has a minimum value of 0.007% and a maximum value of 0.209%, and equation 16 has a minimum value of -0.009% to a maximum value of 0.008%. The equation that has the best fit in descending order is equation 16, 11 and 15, this is because equations 16 and 11 have all the terms of the solution of the cubic equation (19). However, the overall error of the three equations ranges from -0.009 to 0.209% (Table 6).

This range of errors is smaller than that determined by Baiamonte (2015), who presented a procedure of four implicit equations to design irrigation lines of emitters with relative errors smaller than 2% and Baiamonte (2017), who simplified an analytical procedure for the optimal design of irrigation lines of emitters with errors lower than 1.9%. On the other hand, Monge *et al*. (2019) determined energy losses in a multi-gate bamboo conduction pipe using the Darcy & Wesbach, Manning and Hazen & Williams equations and determined that the Darcy & Wesbach and Manning equations better estimate energy losses for rough pipes. Similarly, Taş and Ağıralioğlu (2018) calculated the energy losses in pipes and determined that the Darcy & Wesbach and Manning equations are the best.

Table 6. Values of 'N' number of drippers with different equations and relative errors for different internal diameters of PVC pipes.

Internal diameter (mm)	Values of 'N' in the different equations				Relative errors of 'N' (%)		
	19	11	15	16	11	15	16
17.5	229.12	229.1	229.6	229.1	-0.009	0.209	-0.009
27.8	460.35	460.33	460.83	460.33	-0.004	0.104	0.004
29.8	509.82	509.86	510.36	509.86	0.008	0.106	0.008
40.5	795.11	795.09	795.59	795.09	-0.003	0.06	-0.003
43.9	892	892.04	892.54	892.04	0.004	0.061	0.004
50.7	1 093.85	1 093.92	1 094.42	1 093.92	0.006	0.052	0.006
55.1	1 229.82	1 229.8	1 230.3	1 229.8	-0.002	0.039	-0.002
81.5	2 118.86	2 118.83	2 119.33	2 118.83	-0.001	0.022	-0.001
104.9	2 995.35	2 995.32	2 995.82	2 995.32	-0.001	0.016	-0.001
106.7	3 065.76	3 065.73	3 066.23	3 065.73	-0.001	0.015	-0.001
108.1	3 120.79	3 120.77	3 121.27	3 120.77	-0.001	0.015	-0.001
154.5	5 069.87	5 069.84	5 070.34	5 069.84	-0.001	0.009	-0.001
157.3	5194.59	5 194.56	5 195.06	5 194.56	-0.001	0.009	-0.001
159.5	5 284.14	5 284.11	5 284.61	5 284.11	-0.001	0.009	-0.001
201.3	7 247.19	7 247.17	7 247.67	7 247.17	0	0.007	0
204.9	229.12	229.1	229.6	229.1	-0.009	0.209	-0.009
207.9	326.98	326.97	327.47	326.97	-0.003	0.15	-0.003

Data calculated using equations 11, 15, 16 and 19.

According to the results of the authors mentioned before and the results of Table 6, the relative errors of 'N' are acceptable for equations 11, 15 and 16 and can be used in the design of agricultural drip and sprinkler irrigation.

Conclusions

It was possible to generate three deterministic equations based on the Darcy & Wesbach and Manning equations, (one of total solution equation (11) and two compact equations (15) and (16)), for the design of agricultural drip irrigation systems. In the three equations generated, it was observed that as the number of emitters (drippers) increases, the error of the equations tends to zero. The relative error for calculating the number of emitters in different internal diameters of PVC pipeline ranged from -0.009% to 0.209% for the three equations.

Cited literature

Adams, T. M. 2016. Improved method for converting equivalent sand-grain roughness to Hazen-Williams coefficient. Proceedings of the 2nd World Congress on Mechanical, Chemical, and Material Engineering (MCM 16). Paper No. HTFF 119. Budapest, Hungary. 1-6 pp. Doi: 10.11159/htff16.119.

Alegret-Breña, E. and Martínez-Valdés, Y. 2019. Coeficiente de Hazen-Williams en función del número de Reynolds y la rugosidad relativa. La Habana, Cuba. Ingeniería Hidráulica y Ambiental. 40(3):41-55.

Baiamonte, G. 2015. Simple relationships for the optimal design of paired drip laterals on uniform slopes. Palermo, Italy. J. Irrig. Drain. Eng. 142(2):1-9.

Baiamonte, G. 2017. Advances in designing drip irrigation laterals. Palermo, Italy. Agricultural Water Management. Agric. Water Manag.199(2):157-174.

Espinosa-Espinosa, B.; Flores-Magdaleno, H.; Ascencio-Hernández, R. and Carrillo-Flores, G. 2016. Design of a system of farm hydrant with the Turns and Clement methods: technical and economic análisis. Estado de México, México. Terra Latinoam. 34(4):431-440.

Flechas, R. A. 2010. Efecto del uso de la ecuación de Darcy-Weisbach vs la ecuación de Hazen-Williams sobre los costos de diseños optimizados en acueductos, teniendo en cuenta la rugosidad de las tuberías, línea del gradiente hidráulico y temperatura. Bogotá, Colombia. 1-154 p.

Haddad, A. 2019. Evaluation and correlation of friction head losses in smooth and rough pipes. Antalya, Turkey. Eurasia Proceedings of Science, Technology, Engineering & Mathematics. 7(4):357-362.

Hassan, A. A. 2017. Design of sprinkler irrigation tapered main line by linear programming. Cayro, Egypt. Irrig. Drain 34(3):335-1350.

Jamil, R. 2019. Frictional head loss relation between Hazen-Williams and Darcy-Weisbach equations for various water supply pipe materials. Imam, Saudi Arabia. Int. J. Water. 13(4):333-347.

Jamil, R. and Mujeebu, M. A. 2019. Empirical relation between Hazen-Williams and Darcy-Weisbach equations for cold and hot water flow in plastic pipes. Imam, Saudi Arabia. WATER. 108(6):104-114.

Jiménez-Jiménez, S. I. y Ramírez-Ruíz, C. 2018. Diseño hidráulico de tuberías con salidas múltiples mediante métodos numéricos. Aguascalientes, México. COMEII. 1-10 pp.

Melo, V. G. M. L.; Araújo, A. C. S.; Camargo, A. P.; Melo, L. L.; Frizzone, J. A. and Bombardelli, W. W. A. 2019. Head loss in thin-walled drip tapes with continuous labyrinth. Sao Paulo, Brazil. The Sci. World J. 2019(4)1-11.

Monge-Freile, M.; Sánchez-Delgado, M.; Huanca-Velarde, L. A. and Moreno-Llacza, A. 2019. Bamboo (*Guadua angustifolia* spp.) as a conduction alternative for a multi floodgates irrigation system. Lima, Perú. Anales Científicos. 80(1):240-252.

Monserrat, J.; Barragan, J. and Cots, L. 2018. Design of paired laterals on uniformly sloping fields. Lleida, Spain. J. Irrig. Drain. Eng. 144(6):1-14.

Nosrati, K.; Tahershamsi, A. and Taheri, S. H. S. 2017. Numerical analysis of energy loss coefficient in pipe contraction Using ANSYS CFX Software. Tehran, Iran. Civil Eng. J. 3(4):288-300.

Olivares, A.; Rodrigo, G. R.; Marco, A. M.; Notte, C. E. and Puentes, L. 2019. Experimental evaluation of correlations used to calculate friction factor for turbulent flow in cylindrical pipes. Serena. Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería. 35(1):1-6.

Pimenta, B. D.; Robaina, A. D.; Peiter, M. X.; Mezzomo, W.; Kirchner, J. H. and Ben, L. H. B. 2018. Performance of explicit approximations of the coefficient of head loss for pressurized conduits. Santa María, Brazil. Rev. Bras. Eng. Agr. Amb. 22(5):301-307.

Sadeghi, S. H.; Peters, T. and Shelia, V. 2016. Energy grade line assessment for tapered microirrigation laterals. Washington, United States of America. J. Irrig. Drain. Eng. 142(7):1-11.

SARH. 1979. Secretaría de Agricultura y Recursos Hidraúlicos. Funcionamiento hidráulico, diseño y evaluación de sistemas de riego por goteo. Gómez Palacio, Durango, México. boletín 4 Centro Nacional de Métodos Avanzados de Riego (CENAMAR). 1-416 pp.

Taş, E. and Ağıralioğlu, N. 2018. Comparison of friction losses in long polyethylene pipe systems using different formulas. International Symposium on Urban Water and Wastewater Management Proceding. 602-609 pp.

Tas, E.; Agiralioglu, N.; Mehr, A. D. and Tur, R. 2020. Energy loss investigation in submarine pipelines: case study of cyprus water supply project. Antalya, turkey. Adv. Res. Civil Eng. 2(2):31-44.

Vilaça, F. N.; de Camargo, A. P.; Frizzone, J. A.; Mateos, L. and Koech, R. 2017. Minor losses in start connectors of microirrigation laterals. Sao Paulo, Brazil. Irrig. Sci. 35(3):227-240.

Wang, J. and Chen, R. 2019. An improved finite element model for the hydraulic analysis of drip irrigation subunits considering local emitter head loss. Jiangsu, China. Irrig. Sci. 38(17)1-16.

Wang, J.; Yang, T.; Wei, T.; Chen, R. and Yuan, S. 2019. Experimental determination of local head loss of non-coaxial emitters in thin-wall lay-flat polyethylene pipes. Jiangsu, China. Biosyst. Eng. 190(5):71-86.

Wang, J.; Zhu, D.; Bralts, V. F. and Zhang, L. 2016. Hydraulic analysis of looped microirrigation submain units using the finite element method. Yangling, China. American Society of Agricultural and Biological Engineers. 59(3):909-923.

Wang, Y.; Zhu, D.; Zhang, L. and Zhu, S. 2018. Simulation of local head los in trickle lateral lines equipped with in-line emitters based on dimensional analysis. Yichang, China. Irrigation and Drainage. Irrig. Sci. 67(4)1-10.