Investigation note

Economic optimum in a bivariate Cobb-Douglas function: an application to semi-extensive beef farming

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Abstract

The objective of this research is to determine the economic optimum when the production technology is such that the marginal product of an input is not separable from the use of other inputs. With field information from 51 producers of beef cattle under semi-extensive conditions, surveyed during February-July 2015, in the south of the State of Mexico and obeying a production simplification to two inputs to avoid expressions of marginal products not analytically treatable, the economic optimum for two inputs was obtained, concluding that the intensity of use of this optimum is greater than 30% of the average use in the sample.

Keywords: beef cattle, economic optimum, net income, semi-extensive.

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In Mexico, the production of beef cattle is one of the economic activities of great importance for this sector. During 2006-2015, carcass beef production grew at a AAGR of 1%, going from 1 550 thousand t in 2006 to 1 668.4 in 2015 (FAPRI, 2016), an activity led by Veracruz, Jalisco and Chiapas (SIAP, 2016). In 2015, the apparent national consumption (ANC) of this meat was 2 180.2 thousand t; of this total 76.5% (1 668.4 thousand t) was domestic production and 23.5% (511.8 thousand t) imports (FAPRI, 2016). By state, Veracruz ranked first with 13.5%, while the State of Mexico ranked 18th with 2.5% (85.9 thousand t) of the total volume (SIAP, 2016).

The diversity of production systems from which the main product comes, which is beef, allows generating jobs that strengthen the economic livelihood of thousands of families that depend on it (Puebla *et al*., 2015; Callejas *et al*., 2017). In Mexico, the production of beef animals is obtained, mainly, from semi-extensive systems based on grazing and supplementation in facilities, located in the southeast and central western Mexico, where agroclimatic conditions favor their permanence (García *et al*., 2014); also, from cow-calf systems, stabled or in confinement, dual purpose and small-scale production (Rojo *et al*., 2009; Albarrán *et al*., 2014; Albarrán *et al*., 2015).

The south of the State of Mexico is made up of the municipalities of Temascaltepec, San Simón de Guerrero, Tejupilco, Luvianos, Amatepec and Tlatlaya, grouped in the Rural Development District (DDR, for its acronym in Spanish) 076, characterized by producing beef cattle, but the most important by their volume are Tlatlaya (30%), Amatepec (20%), Luvianos (16%) and Tejupilco (15%) (SIAP, 2016).

In this region, the production of beef animals, with breeds such as Gyr, Nelore, Brahaman and Guzerat (*Bos indicus*), comes from a semi-extensive dual-purpose system (DP) under dry tropical conditions (Puebla *et al*., 2015; Vences *et al*., 2015), with predominance in beef and milk under grazing (rainy season) (Puebla *et al*., 2015; Vences *et al*., 2015) and intensive finishing in corral (dry season) (Puebla *et al*., 2015; García *et al*., 2014; Vences *et al*., 2015).

The agroclimatic conditions of the region and the production system allow the use of resources, under use schemes such as fixed inputs and variable inputs and determine both the state of the art and the technology used by producers (García *et al*., 2014; Albarrán *et al*., 2014). The problem of how much to use an input has to do with the price of the product, the price of the inputs used and feed conversion (marginal productivity). Here, the use of inputs is explored, contrasting the work of the producer relative to a maximum profit approach.

The objective of this research is to estimate the economic optimum, applied to the beef cattle production system under semi-extensive conditions and to determine the quantity of variable inputs that correspond to the maximization of net income. The central hypothesis assumes that the level of input used differs from the optimal economic value.

The fieldwork was carried out from February to July 2015 and considered information from beef cattle producers, located in the municipality of Luvianos, State of Mexico, belonging to DDR 074 of SAGARPA. A direct semi-structured survey was applied, by means of mixed sampling; that is, selective or intentional sampling (Cochran, 1984) and snowball (Joseph, 2009), in such a way that one producer recommended another and the latter to another and so on, until completing a sample of 51 livestock partners, out of a total of 400, belonging to the Regional Livestock Union of the South of the State of Mexico.

The surveys were applied during monthly meetings with livestock partners who attended and were willing to provide information on livestock activity, which is why it was not considered a probabilistic sampling; however, given the similarity between producers it was considered representative. Each producer willing to respond helped fill out their own survey.

The variables that were captured in the survey and subsequently used for the model were: yield in beef (from *in vivo* cattle), whose information was generated directly from the producer at the time of the survey; number of hectares of natural pasture (which included any type of pasture that the producer had on the land surface, paddock or range, where they grazed their animals at the time of the study) and feed consumption (data provided by the livestock producer at the time of the survey).

The information was captured and sorted in an Excel spreadsheet, classified according to the structure of the model. For the fit of a productive technology, the Cobb-Douglas function was used, because this function is a first-order logarithmic approximation to any arbitrary technology (Reynes, 2019). This function was fitted to a set of data obtained in the field with information on the production of live cattle and two variable inputs: natural forage and feed consumption. The simplification to two inputs is to avoid high nonlinearity.

Since the method uses the first derivative, if a third input is used, this derivative is an expression that may result in nonconvergence to a solution in the search for the economic optimum, although beef production considers more than two inputs, the results presented here can be thought of as conditional on the use of inputs not considered. Then, the production function model used is.

$$Y= \theta_0 X_1^{\theta_1} X_2^{\theta_2} + U \qquad 1)$$

This model (1) is a bivariate Cobb-Douglas production function (Castellanos *et al*., 2006; Gujarati, 2010), that is, with two variable inputs, and additive error, such that $E(U)= 0$ and $E(UU')= \sigma^2 I$. This function can be transformed, so that the transformed model is linear in the parameters, with multiplicative error, however, such a multiplicative error would imply a metric in distance to the expected behavior large and implausible (Prajneshu, 2008); therefore, it was estimated as a nonlinear regression model, whose unknowns are the vector $\theta= (\theta_0, \theta_1, \theta_2)'$. Where: Y= total beef production, in tonnes (t); θ_0= scale parameter; X_1= natural forage (kg of solid matter) X_2= feed consumption (kg); θ_1= partial elasticity of production with respect to natural forage input, *ceteris paribus*; θ_2= partial elasticity of production with respect to feed consumption, *ceteris paribus*; U = random error.

The fit of (1) was found with nonlinear least squares (NLLS) (Gallant, 1987). On the other hand, if it is assumed that the orientation of the producer is to maximize their profit, then the objective function to maximize is: $\pi=P_y Y-P_{x_1} X_1-P_{x_2} X_2$ 2). Where: π the monetary profit; P_y is the price of the product (31 000.00 $/t of beef); P_{x_1} the price of input one (25 000.00 $ ha^{-1} of natural forage, estimated as an opportunity cost for the region); and P_{x_2} the price of feed (14 000.00 $ $animal^{-1}$). The prices used are an average of the information provided directly by producers in the survey.

When rewriting (2), the objective function to maximize is: $\pi = P_y(\theta_0 X_1^{\theta_1} X_2^{\theta_2}) - P_{x_1}X_1 - P_{x_2}X_2$ 3).

Thus, after fitting (1), one has the estimated production function as: $Y = \hat{\theta}_0 X_1^{\hat{\theta}_1} X_2^{\hat{\theta}_2}$ 4). Where: $\hat{\theta}_0$, $\hat{\theta}_1$ and $\hat{\theta}_2$ are nonlinear least squares estimators (NLLS). When considering equations (3) and (4), the maximization condition is:

$$\frac{\partial\pi}{\partial X_1} = P_y\left(\hat{\theta}_0\,\hat{\theta}_1\, X_1^{\hat{\theta}_1-1} X_2^{\hat{\theta}_2}\right) - P_{x_1} = 0$$
$$\frac{\partial\pi}{\partial X_1} = P_y\left(\hat{\theta}_0\,\hat{\theta}_2\, X_1^{\hat{\theta}_1} X_2^{\hat{\theta}_2-1}\right) - P_{x_2} = 0$$

5). Where in principle, the optimal intensity of use of inputs is what is sought; that is, the surveyed data of inputs (X_1, X_2) were used to estimate the parameters of the production function, which yielded the estimated vector ($\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2$), this information was used to find the optimal intensity of use (X_1^*, X_2^*). For this, the system of equations proposed in 5 was solved, replacing the relevant price information (P_y= price of the product and the price of inputs P_{x_1}, P_{x_2}).

A problem that arises is that such a system is also nonlinear and interdependent, which prevents an analytical solution of the economic optimum; therefore, a numerical solution with the Gauss-Newton algorithm is used (Torres *et al*., 2020).

Implicitly, the system of equations was as follows: $\begin{bmatrix} F_1\ (X_1, X_2, \theta) \\ F_2\ (X_1, X_2, \theta) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Gauss-Newton consists of replacing the system stated in 5, with a linear or first-order approximation in Taylor expansion (6) around an initial value, for example X_1^I and X_2^I (Torres *et al*., 2020) as follows:

$$\begin{bmatrix} F_1\ (X_1, X_2, \theta) \\ F_2\ (X_1, X_2, \theta) \end{bmatrix} = \begin{bmatrix} F_1\ (X_1^I, X_1^I, \theta) \\ F_2\ (X_1^I, X_1^I, \theta) \end{bmatrix} + \begin{bmatrix} \frac{\partial F_1\ (X_1^I, X_2^I, \theta)}{\partial X_1} & \frac{\partial F_1\ (X_1^I, X_2^I, \theta)}{\partial X_2} \\ \frac{\partial F_2\ (X_1^I, X_2^I, \theta)}{\partial X_1} & \frac{\partial F_2\ (X_1^I, X_2^I, \theta)}{\partial X_2} \end{bmatrix} \begin{bmatrix} X_1^I - X_1 \\ X_2^I - X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

6). The idea is to solve the approximate system as:

$$\begin{bmatrix} X_1^* \\ X_2^* \end{bmatrix} = \begin{bmatrix} X_1^I \\ X_2^I \end{bmatrix} - \begin{bmatrix} \frac{\partial F_1\ (X_1^I, X_2^I, \theta)}{\partial X_1} & \frac{\partial F_1\ (X_1^I, X_2^I, \theta)}{\partial X_2} \\ \frac{\partial F_2\ (X_1^I, X_2^I, \theta)}{\partial X_1} & \frac{\partial F_2\ (X_1^I, X_2^I, \theta)}{\partial X_2} \end{bmatrix}^{-1} \begin{bmatrix} F_1\ (X_1^I, X_1^I, \theta) \\ F_2\ (X_1^I, X_1^I, \theta) \end{bmatrix}$$

7).

As X_1^* and X_2^* solve the approximate system, these values can be used for a next iteration and obtaining a solution. Such a solution can be selected from a convergence criterion, for example, when the Euclidean distance of the vector X_{t+1}^* and X_t^* is less than some small ε (for example 10^{-4}). To estimate the Cobb-Douglas type production function, the Model procedure of the Statistical Analysis System (SAS), version 9.4 (SAS, 2009), was used.

The fit by nonlinear least squares is shown in Table 1; it is noteworthy that the estimators of the elasticities were highly significant, but not the scale parameter (θ_0).

Table 1. Estimators of the Cobb-Douglas production function.

Parameter	Estimator	Standard deviation	Prob. value
θ_0	5.858285	8.7475	0.5063
θ_1	0.318609	0.0682	0.0001
θ_2	0.488534	0.1458	0.0016

With the results obtained, the estimated production function is written as: $Y= 5.858 \cdot X_1^{0.319} X_2^{0.489}$. The economic optimal level maximizes the profit in money (Rebollar *et al*., 2016), the mathematical condition for the economic optimal level (EOL) in a nonlinear model is indicated by the maximization conditions (5).

In the estimation of the economic optimum, the prices used were: P_y= 31 000; P_{x_1}= 25 000; P_{x_2}= 14 000. To find the economic optimum, the Gauss-Newton iterations of the system proposed in (7) began at the arithmetic mean of the use of each input (196 352 and 8 572). The optimal level of X_1 was 724 399 620 (natural forage) and that of X_2 of 8 0781 566 (feed consumption). It was observed that the level of the economic optimum, for these two variable inputs, occur at high levels.

This result is due to the fact that all the beef production obtained is attributed only to these two inputs, which is only an abstraction; this implies that the total cost of production is underestimated and therefore, the algorithm recommends using the inputs in greater quantity. The result obtained, however, contributes to the illustration of estimating economic optimums when no analytical solution can be obtained due to the nonlinearity and non-separability of the marginal product, the way to solve the problem is adequate and without the numerical approach, it is impossible to give any policy recommendation.

Regarding the validation of the model, the dependent variable was tested for normality under the Shapiro-Wilk and Kolmogorov tests (Table 2), for 51 observations. Both tests resulted in not rejecting the normal distribution given the probability values of 0.2 and 0.55 respectively.

Table 2. Normality test.

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	*p*-value	Statistic	df	*p*-value.
Y	0.034	50	0.2*	0.993	50	0.558

*= this is a lower limit of true significance. a. Lilliefors significance correction. d. f.= degrees of freedom.

The exponents of the estimated Cobb-Douglas function add up to 0.81. This result indicates that beef production in the study region is in a situation of diminishing returns to scale (Chiang, 2006). With this result, a 1% increase in the level of use of inputs: natural forage and feed (prepared or commercial), it is expected that the beef yield, in the short term, will increase by less than 1%.

In this regard, in a finding (Acs, 1990), they indicated that farmers take advantage of advantages derived from their volume of beef production, so such a result would show that the efficiency achieved by them is due, in large part, to the efficiency in the management of their resources and the learning they have achieved with many years of production (learning by doing), because, as stated in other studies (Nguyen and Lee, 2012), they are factors in determining the growth of livestock enterprises in the region.

According to the elasticities of the model, for every 1% increase in the level of use of natural forage (NF) by farmers, it is expected, under the conditions proposed, that beef production will increase by 0.32%, *ceteris paribus*. Similarly, for every 1% increase in feed consumption (commercial or prepared) that the producer offers to their animals, *ceteris paribus*, it is expected that, under the conditions proposed, total beef production will increase by just under 1% (0.49).

In a similar work (Pech *et al*., 2002) for Yucatan, Mexico, a production function for beef and milk was applied, it was concluded that it is not necessary to increase feed consumption to increase beef and milk production, but it is necessary to increase the number of animals and labor, in addition to adequate breeding and genetic improvement programs.

Conclusions

When a Cobb-Douglas production function with two or more inputs is fitted, the economic optimum may not exist analytically. However, with a numerical adjustment, the Gauss-Newton method allowed obtaining a numerical estimate of the economic optimum to be able to give economic recommendations. With the data used, the optimal economic dose of the inputs studied is high, this implies that if only these two inputs were used, a profit would be obtained in the production of live cattle.

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